Deutsches
Forschungszentrum
für Künstliche
Intelligenz GmbH

Research
Report
RR-93-29

Representing Belief in Multi-Agent Worlds via Terminological Logics

Armin Laux

September 1993

Deutsches Forschungszentrum für Künstliche Intelligenz
GmbH

Postfach 20 80
67608 Kaiserslautern, FRG
Tel.: (+49 631) 205-3211/13
Fax: (+49 631) 205-3210

Stuhlsatzenhausweg 3
66123 Saarbrücken, FRG
Tel.: (+49 681) 302-5252
Fax: (+49 681) 302-5341

Deutsches Forschungszentrum für Künstliche Intelligenz

The German Research Center for Artificial Intelligence (Deutsches Forschungszentrum für Künstliche Intelligenz, DFKI) with sites in Kaiserslautern and Saarbrücken is a non-profit organization which was founded in 1988. The shareholder companies are Atlas Elektronik, Daimler-Benz, Fraunhofer Gesellschaft, GMD, IBM, Insiders, Mannesmann-Kienzle, SEMA Group, and Siemens. Research projects conducted at the DFKI are funded by the German Ministry for Research and Technology, by the shareholder companies, or by other industrial contracts.

The DFKI conducts application-oriented basic research in the field of artificial intelligence and other related subfields of computer science. The overall goal is to construct *systems with technical knowledge and common sense* which - by using AI methods - implement a problem solution for a selected application area. Currently, there are the following research areas at the DFKI:

- ❑ Intelligent Engineering Systems
- ❑ Intelligent User Interfaces
- ❑ Computer Linguistics
- ❑ Programming Systems
- ❑ Deduction and Multiagent Systems
- ❑ Document Analysis and Office Automation.

The DFKI strives at making its research results available to the scientific community. There exist many contacts to domestic and foreign research institutions, both in academy and industry. The DFKI hosts technology transfer workshops for shareholders and other interested groups in order to inform about the current state of research.

From its beginning, the DFKI has provided an attractive working environment for AI researchers from Germany and from all over the world. The goal is to have a staff of about 100 researchers at the end of the building-up phase.

Friedrich J. Wendl
Director

Representing Belief in Multi-Agent Worlds via Terminological Logics

Armin Laux

DFKI-RR-93-29

This work has been supported by a grant from The Federal Ministry for Research and Technology (FKZ IWT-9201).

Representing Belief in Multi-Agent Worlds via Terminological Logics

Armin Laux
German Research Center for Artificial Intelligence (DFKI)
Stuhlsatzenhausweg 3
66123 Saarbrücken, Germany
e-mail: laux@dfki.uni-sb.de

Abstract

In multi-agent systems a group of autonomous intelligent systems, called agents, acts and cooperates in a world in order to achieve certain goals. Such systems are in general assumed to have no central control structure and hence each agent can only perform actions that are based on his local knowledge and on his local beliefs. In the literature knowledge of agents is mostly represented under the view that knowledge is true belief. On the other hand, if agents are acting in a (real) world their knowledge often is obtained by perception and communication, and hence typically is not true. Thus, the use of belief—where agents may have false beliefs—seems more appropriate than the use of knowledge in multi-agent systems.

Terminological logics provide a well-investigated and decidable fragment of first-order logics that is much more expressive than propositional logic and well suited to describe a world agents are acting in. However, knowledge or belief of agents can only be represented in a very limited way. In this paper we investigate how terminological logics can be extended in such a way that belief of agents can be represented in an adequate manner. We therefore exemplarily extend the concept language $\mathcal{ALC}$ by a modal operator $\Box$, which is indexed by agents. Thereby, $\Box_i\varphi$ represents the fact "agent i believes φ". This belief operator will be interpreted in terms of possible worlds using the well-known modal logic KD45.

This extended language $\mathcal{ALC}_\mathcal{B}$ provides a uniform formalism to describe both, a world agents are acting in and the beliefs agents have about this world and about their own and other agents' beliefs. Thus, it can be seen as a two-dimensional extension of $\mathcal{ALC}$ which allows both, reasoning about objective facts that hold in the world and reasoning on the level of possible worlds. We will give sound and complete algorithms to check consistency of the represented beliefs and to decide whether an $\mathcal{ALC}_\mathcal{B}$-sentence is logically entailed by the beliefs of agents. Hence, when acting in a world agents can use beliefs which are explicitly represented as well as implicit beliefs that are entailed by their knowledge base.

Contents

1 Introduction **3**

2 Syntax and Semantics of $\mathcal{ALC}_\mathcal{B}$ **5**

2.1 The Concept Language $\mathcal{ALC}$ 5

2.2 The Extended Language $\mathcal{ALC}_\mathcal{B}$ 7

3 Testing Satisfiability of $\mathcal{ALC}_\mathcal{B}$-formulas **9**

3.1 The $\mathcal{ALC}_\mathcal{B}$ Frame Algorithm 9

4 Optimization and Computing $\mathcal{ALC}_\mathcal{B}$-Inferences **20**

4.1 Computing $\mathcal{ALC}$ test sets 21

4.2 Restricted $\mathcal{ALC}$-TBoxes 28

4.3 Computing $\mathcal{ALC}_\mathcal{B}$-Inferences 30

5 Conclusion **31**

1 Introduction

Research on the field of multi-agent systems deals with the question how a group of autonomous intelligent systems, called agents, can cooperate in order to achieve certain goals (see, e.g., [6, 15]). As an example, a forwarding agent a and a shipping agent b may cooperate in order to carry out overseas transportation orders.

Although the tasks that multi-agent systems are required to perform are normally stated in terms of the global behavior of the system, the actions that an agent performs can depend only on his local knowledge and on his local beliefs. Thus, there is a close relationship between knowledge, belief, and action in multi-agent systems (see, e.g., [27, 28, 16]). Suppose, in the above example agent a wants to offer a price for carrying out some transportation order o_1. If he believes that there is no other forwarding agent who also can carry out o_1, he will most likely offer another price as in the case where he believes that there is a competitor for this order. And if he even *knows* that there is no competitor for this order, he perhaps offers an exorbitant price. In a recent paper [21] we investigated how knowledge of agents can be represented on the basis of terminological logics, whereby we used the classical view of knowledge as true belief. That means, an agent knows φ if he believes that φ holds and φ actual does hold. On the other hand, as pointed out in, say [23], the knowledge represented in a knowledge base typically is not true. Thus, the use of beliefs—where agents may have false beliefs—seems more appropriate than knowledge for formalizing the reasoning and deduction of a knowledge base. In the current paper we concern with the question how agents can be equipped with beliefs about the world they are acting in, about beliefs of other agents, and also about their own beliefs. Thereby, it should be taken into consideration that different agents may have different beliefs about the same notions. For example, forwarding agent a may believe that *company XY* is a rich company and a good client, while forwarding agent B believes that *company XY* is rich but not a good client. Using the language $\mathcal{ALC}_\mathcal{B}$, which is presented in the next section, this can be formalized by

$$\Box_a(\textit{company XY}\text{: rich-company} \sqcap \text{good-client}) \quad \text{and}$$
$$\Box_b(\textit{company XY}\text{: rich-company} \sqcap \neg\text{good-client})$$

respectively, where $\Box_i\varphi$ is to be read as "agent i believes φ".

Since the work of Hintikka [18], modal logics have widely been accepted to be an adequate formalism for representing knowledge and belief of agents. The intuitive idea here is that besides the real world agents can imagine a number of other worlds (situations) to be possible. By imposing various conditions on this possibility relation, we can capture a number of interesting axioms. For example, if we require that the world that the agent finds himself in is always one of the worlds he considers possible (which amounts to saying that the possibility relation is reflexive), then it follows that the agent does not know false facts. When using a possibility relation which captures axioms of knowledge (belief) an agent is said to know (believe) a fact φ iff φ is true in all worlds he thinks to be possible. For example, an agent knows (believes) that there exists a monster of Loch Ness if there is such a monster in all worlds he considers possible. To express the beliefs of an agent a in this approach a binary operator *BELIEF* (a, φ) is used, where φ is a formula over some logical language $\mathcal{L}$. If we want to devise a formalism for representing the beliefs of agents we have to take two decisions. Firstly, we have to decide what the general properties of belief are we want this formalism to capture. Secondly, we have to choose a suitable logical representation language $\mathcal{L}$ which allows to describe the beliefs of agents.

There are many approaches to determine axioms characterizing belief (see, e.g., [22, 28, 24, 25, 26, 12, 17]). We will use the following axiomatization which has been most commonly used in the literature. The first of these properties states that an agent does not believe false facts. That means, an agent cannot believe both a fact and its negation, though he can believe facts which actually do not hold in the world. Secondly, if an agent believes a fact then he believes that he believes it (positive introspection), and if he does not believe in a fact then he believes that he does not believe in it (negative introspection). From this it follows, e.g., that agents believe that their beliefs are true (weak reflexivity). Finally, the probably most important property is that agents can reason on the basis of their beliefs. For example, suppose agent a believes that each truck which is owned by John can be used to transport gasoline and he believes that John owns the truck *truck-1*. In this case, agent a must be able to conclude that John's truck *truck-1* can (probably) be used to transport gasoline, and thus may negotiate with John for a transportation order.

As logical language to describe belief of agents we will use a terminological logic. Terminological logics provide a well-investigated and decidable fragment of first-order logics that is much more expressive than propositional logic. They are based on the work of Brachman and Schmolze [9] and have been developed as a structured formalism to describe the relevant concepts of a problem domain and the interactions between these concepts. Starting with atomic concepts (unary predicates) and roles (binary predicates), one therefore defines complex concepts with the help of operators provided by a concept language, and interactions between (complex) concepts are expressed by a set of so-called terminological axioms. On the other hand, by so-called assertional axioms, objects can be associated with concepts and relationships between objects can be defined via roles. For example, we can use these logics to represent facts like "each truck which is owned by John can be used to transport gasoline" or "John owns *truck-1* which is a truck".

In the literature, a lot of concept languages have been considered (see, e.g., [8, 29, 3]). But they all have in common that they are only suitable for representing objective facts about the world, and knowledge or beliefs of agents can only be represented in a very limited way. Thus, we need an extended concept language which allows the representation of belief according to the above given (informal) axiomatization. Since the work of Schild [31] it is known that the concept language $\mathcal{ALC}$ provides a terminological logic which is a notational variant of the propositional modal logic $\mathsf{K}_{(m)}$. However, it is not investigated there how to extend this logic to a two-dimensional logic which allows reasoning on both the objective level and the level of possible worlds. In order to combine both levels one has to define syntax and semantics of an extended language. Baader and Ohlbach [5] present a multi-dimensional extension of $\mathcal{ALC}$, where multi-modal operators can be used at all levels of the concept terms and they can be used to modify both concepts and roles. However, the underlying logic is simply the basic modal logic K, and it is not yet clear how to extend their approach in such a way that modal logics different from K can be handled. Moreover, they could not succeed in proving completeness of their satisfiability algorithm.

In this paper we will present a different extended language where (sequences of) modal operators are only allowed in front of terminological and assertional axioms. This language allows one to interpret the modal operators w.r.t. modal logics different from K, e.g., S4 (see [21]) or KD45 (in the present paper). This language, called $\mathcal{ALC}_\mathcal{B}$, can be seen as a two-dimensional representation language with terminological and assertional axioms as primitives where each primitive may describe a part of the world and each agent can believe a set of such primitives to hold in the world. The modal operators, which are indexed with agents, are interpreted in terms of possible worlds in such a way that they satisfy the above axiomatization of belief, what amounts in using the modal logic KD45. Thus, the resulting language provides a uniform formalism to describe both, a world agents are acting in as well as the beliefs agents have about this world and about their own and other agents' beliefs. We will give sound and complete algorithms for deciding satisfiability of $\mathcal{ALC}_\mathcal{B}$-formulas and for testing whether an $\mathcal{ALC}_\mathcal{B}$-formula is entailed by a given set of $\mathcal{ALC}_\mathcal{B}$-formulas. Hence, when acting in a world agents can use beliefs which are explicitly represented as well as implicit beliefs that are entailed by their knowledge base.

2 Syntax and Semantics of $\mathcal{ALC}_\mathcal{B}$

In this section we will formally introduce the language $\mathcal{ALC}_\mathcal{B}$ which extends the concept language $\mathcal{ALC}$ by a modal operator $\Box_i$ for each agent. Syntax and semantics of $\mathcal{ALC}$ and $\mathcal{ALC}_\mathcal{B}$ are given in Subsections 2.1 and 2.2, respectively.

2.1 The Concept Language $\mathcal{ALC}$

Terminological logics provide two formalisms to describe a problem domain: a terminological formalism to represent taxonomical knowledge by defining concepts, which can be seen as sets of objects, and an assertional formalism which can be used to describe concrete objects. Therefore, one starts with a set of *atomic concepts* (unary predicates) and a set of *roles* (binary predicates). In the concept language $\mathcal{ALC}$ *concepts* are then built up from atomic concepts, the *top concept* $\top$, the *bottom concept* $\bot$, and roles inductively by:

1. Each atomic concept, $\top$, and $\bot$ are concepts.
2. If C and D are concepts and R is a role, then
   (a) $C \sqcap D$ (*concept conjunction*),
   (b) $C \sqcup D$ (*concept disjunction*),
   (c) $\neg C$ (*concept negation*),
   (d) $\forall R.C$ (*value restriction*), and
   (e) $\exists R.C$ (*exists restriction*)

   are concepts.

An interpretation I is a function over some non-empty domain Δ^I which maps each atomic concept C to a subset C^I of Δ^I, each role R to a subset R^I of $\Delta^I \times \Delta^I$, $\top$ to Δ^I, and $\bot$ to $\emptyset$. Furthermore, $\sqcap$ is interpreted as set intersection, $\sqcup$ as set union, and $\neg$ as set complement w.r.t. Δ^I. The value and the exists restrictions are interpreted by

$$[\forall R.C]^I = \{d \in \Delta^I \mid \forall d' : (d, d') \in R^I \rightarrow d' \in C^I\}$$
$$[\exists R.C]^I = \{d \in \Delta^I \mid \exists d' : (d, d') \in R^I \wedge d' \in C^I\}$$

For example, if *man* and *truck* are atomic concepts and *owns* is a role we can define the concept of men who own a truck by $man \sqcap \exists owns.truck$.

The taxonomical knowledge of a problem domain can be defined by an $\mathcal{ALC}$-*TBox* (*terminology*), which consists of a finite set of terminological axioms. A *terminological axiom* is of the form

- $C = D$ (concept equivalence) or
- $C \neq D$ (negated concept equivalence)

where C, D are concepts. An interpretation I satisfies $C = D$ iff $C^I = D^I$ and it satisfies $C \neq D$ iff $C^I \neq D^I$. An interpretation I satisfies an $\mathcal{ALC}$-TBox $\mathcal{T}$ iff I satisfies each axiom in $\mathcal{T}$. For example, if *carrier*, *person*, and *truck* are concepts and *owns* is a role, we can define exactly the persons who own a truck to be a carrier by

$$carrier = person \sqcap \exists owns.truck.$$

The assertional formalism of $\mathcal{ALC}$ allows one to introduce concrete objects by stating that they are instances of concepts and roles: If a is an object and C a concept, then $a : C$ is a *concept instance*. If a and b are objects and R is a role, then aRb is a *role instance.* Concept instances and role instances are called *assertional axioms*, and a finite set of assertional axioms is called an $\mathcal{ALC}$-*ABox*. An interpretation I maps objects to elements of its domain Δ^I and satisfies $a : C$ iff $a^I \in C^I$, and aRb iff $(a^I, b^I) \in R^I$. We assume that different objects in an ABox are mapped to different elements in Δ^I (*unique name assumption*). An interpretation I satisfies an $\mathcal{ALC}$-ABox $\mathcal{A}$ iff I satisfies each axiom in $\mathcal{A}$. As an example, if *John* and *truck-1* are objects, we can express that John owns truck-1 which is a truck by the assertional axioms

John owns truck-1 and *truck-1 : truck.*

Thus, we can describe the relevant concepts of a problem domain by terminological axioms, i.e., by an $\mathcal{ALC}$-TBox, and properties of objects as well as relations between them by assertional axioms, i.e., by an $\mathcal{ALC}$-ABox. We say an interpretation I satisfies a set $Ax_1, \ldots, Ax_n$ of terminological and assertional axioms iff I satisfies each of these axioms. We then write $I \models Ax_1, \ldots, Ax_n$.

For sake of simplicity we will sometimes use the expressions $C \sqsubseteq D$ and $C \not\sqsubseteq D$ where C and D are concepts. An interpretation I satisfies $C \sqsubseteq D$ iff $C^I \subseteq D^I$ and it satisfies $C \not\sqsubseteq D$ iff $C^I \not\subseteq D^I$. The next lemma states that these expressions are abbreviations for certain terminological axioms.

Lemma 2.1 *Let C and D be concepts, and let I be an interpretation. Then*

1. *I satisfies $C \sqsubseteq D$ iff I satisfies $\neg C \sqcup D = \top$.*

2. *I satisfies $C \not\sqsubseteq D$ iff I satisfies $\neg C \sqcup D \neq \top$.*

Proof: For 1., firstly suppose I satisfies $C \sqsubseteq D$. Then for each element d in Δ^I either $d \in [\neg C]^I$ or both $d \in C^I$ and $d \in D^I$ holds. That means, I satisfies $\neg C \sqcup (C \sqcap D) = \top$ what can be simplified to $\neg C \sqcup D = \top$. Conversely, suppose I satisfies $\neg C \sqcup D = \top$. Then for each element $d \in \Delta^I$ either $d \notin C^I$ or $d \in D^I$ holds. Thus, from $d \in C^I$ follows $d \in D^I$, i.e., $C^I \subseteq D^I$. The proof of 2. is analogous. □

For example, if *truck* and *vehicle* are concepts we can define each truck to be a vehicle by *truck* $\sqsubseteq$ *vehicle*, what is an abbreviation for $\neg$*truck* $\sqcup$ *vehicle* $= \top$.

2.2 The Extended Language $\mathcal{ALC}_\mathcal{B}$

Now we will introduce the language $\mathcal{ALC}_\mathcal{B}$ which extends $\mathcal{ALC}$ by a new operator $\Box_i$ for each agent i.[1] We allow these operators in front of terminological and assertional

[1] In the following, we will abbreviate agents by numbers, and we suppose only a finite number of agents to be given.

axioms. Thereby, the operator $\Box_i$, read as "agent i believes", allows us to express the beliefs agent i has about the world, about beliefs of other agents, and about his own beliefs. We extend the definition of terminological and assertional axioms as follows.

- If *TA* is a terminological axiom, then $\Box_i$ *TA* and $\neg\Box_i$ *TA* are terminological axioms as well.
- If *CI* is a concept instance, then $\Box_i$ *CI* and $\neg\Box_i$ *CI* are concept instances as well.
- If *RI* is a role instance, then $\Box_i$ *RI* is a role instance as well.

Note, that we do not allow formulas of the form $\neg\Box_i(aRb)$. The reason for this restriction is that such axioms would be equivalent to stating that there exists a world in which the role instance aRb does *not* hold. And negation of roles is not allowed in $\mathcal{ALC}$.

These extended assertional and terminological axioms are called *$\mathcal{ALC}_\mathcal{B}$-formulas* and can, e.g., be used to state that agent i believes that each truck is a vehicle by

$$\Box_i\ (\mathit{truck} \sqsubseteq \mathit{vehicle}).$$

Analogously, the $\mathcal{ALC}_\mathcal{B}$-formulas $\Box_i\neg\Box_j$ (*vehicle-1 : truck*) and $\Box_i\neg\Box_i$ (*vehicle-1 : truck*) are to be read as "agent i believes that agent j doesn't believe that vehicle-1 is a truck" and "agent i believes that he doesn't believe truck-1 to be a truck", respectively. Allowing $\Box_i$ immediately in front of concepts (possibly $\Box_i C$ may be interpreted as "the set of individuals agent i believes to be a C") causes essential algorithmic problems and is out of the scope of this paper.

We will interpret the operators $\Box_i$ in terms of *possible worlds*, i.e., besides the real world there exist a number of worlds agents consider to be possible. If agent i considers world w' possible at world w, we say w' is *accessible from* w by agent i. The *accessibility relation* of agent i is given by all pairs (w, w') such that w' is accessible from w by agent i. Since different worlds are possible in our approach, the interpretation of concepts and roles in $\mathcal{ALC}_\mathcal{B}$-formulas depends on the world we are currently speaking of. That means, in different worlds concepts may contain different objects and roles may contain different pairs of objects. This will be expressed by taking an additional parameter, the *world parameter*, into consideration when interpreting concepts and roles. Formally, we use the notion of a *K-interpretation* K_I which consists of a non-empty domain Δ^{K_I} and maps objects to elements in Δ^{K_I} while satisfying the unique name assumption, atomic concepts to subsets of $\Delta^{K_I} \times \mathcal{W}$, $\top$ to $\Delta^{K_I} \times \mathcal{W}$, $\bot$ to $\emptyset \times \mathcal{W}$, and roles to subsets of $\Delta^{K_I} \times \Delta^{K_I} \times \mathcal{W}$. Furthermore, $\sqcap$ is interpreted as set intersection, $\sqcup$ as set union, and $\neg$ as set complement w.r.t. $\Delta^{K_I} \times \mathcal{W}$, and the value and exists restrictions are interpreted by

$$[\forall R.C]^{K_I} = \{(d, w) \mid (d', w) \in C^{K_I} \text{ for each } d' \text{ with } (d, d', w) \in R^{K_I}\}$$
$$[\exists R.C]^{K_I} = \{(d, w) \mid (d', w) \in C^{K_I} \text{ for some } d' \text{ with } (d, d', w) \in R^{K_I}\}.$$

Definition 2.2 *A Kripke structure K is a triple $(\mathcal{W}, \Gamma, K_I)$. Thereby, $\mathcal{W}$ is a non-empty set of worlds, Γ is a finite set of accessibility relations, one accessibility relation γ_i for each agent i, and K_I is a K-interpretation.*

The *satisfiability* of an $\mathcal{ALC}_\mathcal{B}$-formula F *in a Kripke structure* $K = (\mathcal{W}, \Gamma, K_I)$ *and a world* $w \in \mathcal{W}$, written as $K, w \models F$, is recursively defined by:

$$\begin{array}{lll} K,w \models C = D & \text{iff} & \{d \mid (d,w) \in C^{K_I}\} = \{d \mid (d,w) \in D^{K_I}\} \\ K,w \models C \neq D & \text{iff} & \{d \mid (d,w) \in C^{K_I}\} \neq \{d \mid (d,w) \in D^{K_I}\} \\ K,w \models a : C & \text{iff} & (a,w) \in C^{K_I} \\ K,w \models aRb & \text{iff} & (a,b,w) \in R^{K_I} \\ K,w \models \Box_i G & \text{iff} & K,w' \models G \text{ for each world } w' \text{ with } (w,w') \in \gamma_i \\ K,w \models \neg\Box_i G & \text{iff} & \text{there is a world } w' \text{ with } (w,w') \in \gamma_i \text{ and } K,w' \not\models G \end{array}$$

where G is an $\mathcal{ALC}_\mathcal{B}$-formula, C, D are concepts, a, b are objects, and R is a role.

A set $F_1, \ldots, F_n$ of $\mathcal{ALC}_\mathcal{B}$-formulas is *satisfiable* iff there exists a Kripke structure $K = (\mathcal{W}, \Gamma, K_I)$ and a world $w_0 \in \mathcal{W}$ such that $K, w_0 \models F_i$ for $i = 1, \ldots, n$. We then write $K \models F_1, \ldots, F_n$.

In the following we will use the notion *modality* to denote (negated) indexed $\Box$ operators, and $\mathcal{ALC}_\mathcal{B}$-formulas without any modalities are called $\mathcal{ALC}$*-formulas*. For example, the $\mathcal{ALC}_\mathcal{B}$-formula $\Box_i \neg\Box_j$(*vehicle-1 : truck*) contains the modalities $\Box_i$ and $\neg\Box_j$, and the $\mathcal{ALC}_\mathcal{B}$-formula *vehicle-1 : truck* is an $\mathcal{ALC}$-formula.

3 Testing Satisfiability of $\mathcal{ALC}_\mathcal{B}$-formulas

Using $\mathcal{ALC}_\mathcal{B}$-formulas, a "real world" and belief of agents can be defined as follows. The real world is given by a finite set of $\mathcal{ALC}$-formulas, and the belief of agent i is given by a finite set of $\mathcal{ALC}_\mathcal{B}$-formulas with the leading modality $\Box_i$. Of course, we do not only want to represent a world and beliefs of agents, but we are interested in algorithms to test (i) consistency of the represented facts, i.e., whether a given set of $\mathcal{ALC}_\mathcal{B}$-formulas is satisfiable, and (ii) whether an $\mathcal{ALC}_\mathcal{B}$-formula is a logical consequence of a given set of $\mathcal{ALC}_\mathcal{B}$-formulas. In this section we will give an algorithm for testing satisfiability of a set of $\mathcal{ALC}_\mathcal{B}$-formulas. Building upon this we will show how to decide whether or not an $\mathcal{ALC}_\mathcal{B}$-formula is a logical consequence from a given set of $\mathcal{ALC}_\mathcal{B}$-formulas in Section 4.

3.1 The $\mathcal{ALC}_\mathcal{B}$ Frame Algorithm

We will now present an algorithm for testing satisfiability of a finite set $F_1, \ldots, F_n$ of $\mathcal{ALC}_\mathcal{B}$-formulas. By definition, a set $F_1, \ldots, F_n$ of $\mathcal{ALC}_\mathcal{B}$-formulas is satisfiable iff there exists a Kripke structure K such that $K \models F_1, \ldots, F_n$. Of course, we are not interested in arbitrary Kripke structures to satisfy $F_1, \ldots, F_n$, but only in Kripke structures which interpret the belief operators $\Box$ in $F_1, \ldots, F_n$ in such a way that they satisfy the properties described in Section 1. We therefore introduce the notion of KD45 Kripke structures.

Definition 3.1 *A set $F_1, \ldots, F_n$ of $\mathcal{ALC_B}$-formulas is* KD45-satisfiable *iff there exists a Kripke structure $K = (\mathcal{W}, \Gamma, K_I)$ which satisfies $F_1, \ldots, F_n$ and which has the properties*

(P1)	*if* $K, w \models \Box_i F$ *then* $K, w \models \neg\Box_i\neg F$
(P2)	*if* $K, w \models \Box_i F$ *then* $K, w \models \Box_i\Box_i F$
(P3)	*if* $K, w \models \neg\Box_i F$ *then* $K, w \models \Box_i\neg\Box_i F$

for each $\mathcal{ALC_B}$-formula F, for each agent i, and for each world $w \in \mathcal{W}$.[2] *A Kripke structure which satisfies (P1), (P2), and (P3) is called* KD45 Kripke structure.

Property (P1) corresponds to "an agent cannot believe in both a fact and its negation", (P2) to "if an agent believes something, then he believes that he believes it", and (P3) to "if an agent does not believe in a fact then he believes that he does not believe in this fact". The property "agents must be able to reason on the basis of their beliefs", is guaranteed by choosing Kripke structures for the representation of belief (cf., e.g., [17]).

It is a well-known fact that $K = (\mathcal{W}, \Gamma, K_I)$ is a KD45 Kripke structure if the accessibility relation γ_i of each agent i is serial, Euclidean, and transitive (see, e.g., [25]). A relation $\gamma \subseteq \mathcal{W} \times \mathcal{W}$ is

- *serial* iff for each u in $\mathcal{W}$ there is a v in $\mathcal{W}$ such that $(u, v) \in \gamma$,
- *Euclidean* iff for all u, v, w in $\mathcal{W}$ holds: if $(u, v) \in \gamma$ and $(u, w) \in \gamma$ then $(v, w) \in \gamma$,
- *transitive* iff for all u, v, w in $\mathcal{W}$ holds: if $(u, v) \in \gamma$ and $(v, w) \in \gamma$ then $(u, w) \in \gamma$.

We will use the standard notation $\Diamond_i F$ as an abbreviation for $\neg\Box_i\neg F$ such that the properties (P1) and (P3) can be rewritten as

(P1′)	if $K, w \models \Box_i F$ then $K, w \models \Diamond_i F$
(P3′)	if $K, w \models \Diamond_i F$ then $K, w \models \Box_i\Diamond_i F$.

In the following we will use the one or the other version of these two properties, whichever is more appropriate.

[2]Since these properties hold for *arbitrary* worlds this amounts in saying that all these properties are mutually believed, i.e., each agent's belief has these properties, each agent believes that each agent's belief has these properties and so on.

To keep notation simple we transform $\mathcal{ALC}_\mathcal{B}$-formulas into negation normal form. An $\mathcal{ALC}_\mathcal{B}$-formula (concept) is in *negation normal form* iff in the formula (concept) negation signs occur immediately in front of atomic concepts only. Concepts can be transformed into an equivalent negation normal form by the rules

$$\begin{array}{lll@{\qquad}lll@{\qquad}lll}
\neg\neg C & \to & C & \neg(C \sqcap D) & \to & \neg C \sqcup \neg D & \neg(\forall R.C) & \to & \exists R.\neg C \\
\neg\top & \to & \bot & \neg(C \sqcup D) & \to & \neg C \sqcap \neg D & \neg(\exists R.C) & \to & \forall R.\neg C \\
\neg\bot & \to & \top & & & & & &
\end{array}$$

where C is a concept and R is a role (see, e.g., [20]). Building upon this, $\mathcal{ALC}_\mathcal{B}$-formulas can be transformed into negation normal form by applying the rules

$$\begin{array}{lll@{\qquad}lll}
\neg\neg F & \to & F & \neg(C = D) & \to & a_n : (C \sqcap \neg D) \sqcup (\neg C \sqcap D) \\
\neg\Box_i F & \to & \Diamond_i \neg F & \neg(C \neq D) & \to & C = D \\
\neg\Diamond_i F & \to & \Box_i \neg F & \neg(a : C) & \to & a : \neg C
\end{array}$$

to the outermost negation sign. Thereby, F is an $\mathcal{ALC}_\mathcal{B}$-formula, C, D are concepts, a is an object, and a_n is a new object. For example, the negation normal form of the $\mathcal{ALC}_\mathcal{B}$-formula

$$\neg\Box_i\big(A = \neg(\forall R.C)\big) \quad \text{is} \quad \Diamond_i\big(a_n : (A \sqcap \forall R.C) \sqcup (\neg A \sqcap \exists R.\neg C)\big)$$

where a_n is a new object. The next lemma states that an $\mathcal{ALC}_\mathcal{B}$-formula is KD45-satisfiable iff its negation normal form is KD45-satisfiable.

Lemma 3.2 *Let F be an $\mathcal{ALC}_\mathcal{B}$-formula, F^* be the negation normal form of F, and K be a KD45 Kripke structure. Then $K \models F$ iff there is a KD45 Kripke structure K' such that $K' \models F^*$.*

Proof: If we apply one of the rules $\neg\neg F \to F$, $\neg\Box_i F \to \Diamond_i\neg F$, $\neg\Diamond_i F \to \Box_i \neg F$, or $\neg(a : C) \to a : \neg C$ to an $\mathcal{ALC}_\mathcal{B}$-formula F, then K obviously satisfies the formula on the left hand side of the rule iff K satisfies the right hand side of the rule.

Now suppose $K = (\mathcal{W}, \Gamma, K_I)$ and $K \models \neg(C = D)$, i.e., $K, w_0 \models \neg(C = D)$ for some world w_0 in $\mathcal{W}$. In this case there is an element $u \in \Delta^{K_I}$ such that (u, w_0) is either in C^{K_I} and in $[\neg D]^{K_I}$ or in $[\neg C]^{K_I}$ and in D^{K_I}. Let now $K' = (\mathcal{W}', \Gamma', K_I')$ be a Kripke structure which is identical to K but $a_n^{K_I} := u$. Then, K' is a KD45 Kripke structure[3] and $K', w_0 \models a_n : (C \sqcap \neg D) \sqcup (\neg C \sqcap D)$. Conversely, if K' is a KD45 Kripke structure such that $K', w_0 \models a_n : (C \sqcap \neg D) \sqcup (\neg C \sqcap D)$, then obviously $K', w_0 \models \neg(C = D)$. □

If F is an $\mathcal{ALC}_\mathcal{B}$-formula in negation normal form it has a (possibly empty) leading sequence $\circ^* = \circ_{i_1} \ldots \circ_{i_m}$ of non-negated modalities where each $\circ_{i_j}$ is either □ or ◇ and

[3]Note, that a_n is a *new* element and the unique name assumption only has to hold for objects occurring in an ABox.

each index i_j is an agent. We now replace each subsequence of modalities indexed with the same agent in $\circ^*$ by the last modality in this subsequence. The obtained $\mathcal{ALC}_\mathcal{B}$-formula is called the *KD45 normal form F' of F*. For example, the KD45 normal form of $\Box_1\Diamond_1\Diamond_2\Box_2\Box_1(a : C)$ is given by $\Diamond_1\Box_2\Box_1(a : C)$. As an immediate consequence of Proposition 4.27 in [10], for each KD45 Kripke structure $K = (\mathcal{W}, \Gamma, K_I)$ and for each world $w \in \mathcal{W}$ holds that $K, w \models F$ iff $K, w \models F'$.

Assumption: In the following we suppose each $\mathcal{ALC}_\mathcal{B}$-formula to be in KD45 normal form (and thus especially in negation normal form).

To formulate a calculus for testing KD45 satisfiability of $\mathcal{ALC}_\mathcal{B}$ formulas we introduce the notions of labeled $\mathcal{ALC}_\mathcal{B}$-formulas and of a world constraint system. A *labeled $\mathcal{ALC}_\mathcal{B}$-formula* consists of an $\mathcal{ALC}_\mathcal{B}$-formula F together with a label w, written as $F \,||\, w$. Thereby, w is a constant representing a world in which F holds. A *world constraint* is either a labeled $\mathcal{ALC}_\mathcal{B}$-formula or a term $w \bowtie_i w'$. The constants w and w' represent worlds and $\bowtie_i$ represents the accessibility relation of agent i. A *world constraint system* is a finite, non-empty set of world constraints.

A Kripke structure $K = (\mathcal{W}, \Gamma, K_I)$ satisfies a world constraint system W iff for each label w in W there is a world $w^K \in \mathcal{W}$ such that (i) $K, w^K \models F$ for each world constraint $F \,||\, w$ in W and (ii) $(w^K, v^K) \in \gamma_i$ for each world constraint $w \bowtie_i v$ in W. A world constraint system W is (KD45-)*satisfiable* iff there exists a (KD45) Kripke structure which satisfies W.

For testing KD45-satisfiability of a set $F_1, \ldots, F_n$ of $\mathcal{ALC}_\mathcal{B}$-formulas we firstly translate them into a world constraint system. The world constraint system W is *induced by* $F_1, \ldots, F_n$ iff $W = \{F_1 \,||\, w_0, \ldots, F_n \,||\, w_0\}$, where w_0 is a new constant (which represents the real world). Obviously, $F_1, \ldots, F_n$ are KD45-satisfiable iff W is KD45-satisfiable.

KD45-satisfiability of a world constraint system W is tested by the *frame algorithm* which has a world constraint system as input that is induced by a finite number of $\mathcal{ALC}_\mathcal{B}$-formulas and which successively adds new world constraints to W by applying the four propagation rules in Figure 1. Thus, the result of the frame algorithm with input W is a (modified) world constraint system W'. We will call each world constraint system that can be obtained from W by a finite number of propagation rule applications a *derived system* in the following.

The intuitive idea behind these propagation rules is as follows: Firstly, for $W \rightarrow_\Diamond W'$, if there is a world constraint $\Diamond_i F \,||\, w$ in W we add a world v such that (i) v is accessible from w by agent i and (ii) $F \,||\, v$ holds. Furthermore, whenever $\Diamond_i F_j \,||\, w$ is in W we add $\Diamond_i F_j \,||\, v$ because of property (P'_3), and whenever $\Box_i G_k \,||\, w$ is in W we add both $\Box_i G_k \,||\, v$ and $G_k \,||\, v$ because of property (P_2) and the definition of $\Box_i$. This rule is similar to the unsigned prefixed KD45 tableaux rules in [14]. Secondly, for $W \rightarrow_\Box W'$, if $\Box_i G_1 \,||\, w, \ldots, \Box_i G_m \,||\, w$ are in W but there is no world u accessible from w by agent i, we have to introduce a new world v—accessible from w by agent i—where $G_1 \,||\, v, \ldots, G_m \,||\, v$ and $\Box_i G_1 \,||\, v, \ldots, \Box_i G_m \,||\, v$ holds. This is due to the properties

$W \rightarrow_\Diamond \{w \bowtie_i v, F||v, \Diamond_i F_1||v, \ldots, \Diamond_i F_n||v, G_1||v, \Box_i G_1||v, \ldots, G_m||v, \Box_i G_m||v\} \cup W$

if $\Diamond_i F \,||\, w, \Diamond_i F_1 \,||\, w, \ldots, \Diamond_i F_n \,||\, w$ are the world constraints with leading modality $\Diamond_i$ in W, $\Box_i G_1 \,||\, w, \ldots, \Box_i G_m \,||\, w$ are the world constraints with leading modality $\Box_i$ in W, there is no label u in W such that the world constraints $F \,||\, u$, $\Diamond_i F_1 \,||\, u, \ldots, \Diamond_i F_n \,||\, u$, $G_1 \,||\, u, \ldots, G_m \,||\, u$, $\Box_i G_1 \,||\, u, \ldots, \Box_i G_m \,||\, u$ are exactly the labeled $\mathcal{ALC}_\mathcal{B}$-formulas with label u in W, and v is a new label.

$W \rightarrow_{\Diamond_\circ} \{w \bowtie_i u\} \cup W$

if $\Diamond_i F \,||\, w, \Diamond_i F_1 \,||\, w, \ldots, \Diamond_i F_n \,||\, w$ are the world constraints with leading modality $\Diamond_i$ in W, $\Box_i G_1 \,||\, w, \ldots, \Box_i G_m \,||\, w$ are the world constraints with leading modality $\Box_i$ in W, there is a label u in W such that the world constraints $F \,||\, u$, $\Diamond_i F_1 \,||\, u, \ldots, \Diamond_i F_n \,||\, u$, $G_1 \,||\, u, \ldots, G_m \,||\, u$, $\Box_i G_1 \,||\, u, \ldots, \Box_i G_m \,||\, u$ are exactly the labeled $\mathcal{ALC}_\mathcal{B}$-formulas with label u in W, and $w \bowtie_i u$ is not in W

$W \rightarrow_\Box \{w \bowtie_i v, G_1 \,||\, v, \Box_i G_1 \,||\, v, \ldots, G_m \,||\, v, \Box_i G_m \,||\, v\} \cup W$

if no world constraint of the form $\Diamond_i F||w$ is in W, $\Box_i G_1||w, \ldots, \Box_i G_m||w$ are the world constraints with leading modality $\Box_i$ in W, there is no label u in W such that the world constraints $G_1 \,||\, u, \ldots, G_m \,||\, u$, $\Box_i G_1||u, \ldots, \Box_i G_m||u$ are are exactly the labeled $\mathcal{ALC}_\mathcal{B}$-formulas with label u in W, and v is a new label.

$W \rightarrow_{\Box_\circ} \{w \bowtie_i u\} \cup W$

if no world constraint of the form $\Diamond_i F||w$ is in W, $\Box_i G_1||w, \ldots, \Box_i G_m||w$ are the world constraints with leading modality $\Box_i$ in W, there is a label u in W such that $G_1 \,||\, u, \ldots, G_m \,||\, u$, $\Box_i G_1 \,||\, u, \ldots, \Box_i G_m \,||\, u$ are are exactly the labeled $\mathcal{ALC}_\mathcal{B}$-formulas with label u in W, and $w \bowtie_i u$ is not in W.

Figure 1: Propagation rules of the $\mathcal{ALC}_\mathcal{B}$ frame algorithm.

(P'_1) and (P_2) of KD45 Kripke structures. Finally, the rules $\rightarrow_{\Diamond_\circ}$ and $\rightarrow_{\Box_\circ}$ are used to guarantee termination of applying propagation rules to a derived system.

Now we will show that the $\mathcal{ALC}_\mathcal{B}$ frame algorithm has the following two important properties. Firstly, it terminates for every world constraint system W as input which is induced by a finite set $F_1, \ldots, F_n$ of $\mathcal{ALC}_\mathcal{B}$-formulas. Secondly, if W' is the result of the frame algorithm with input W, then $F_1, \ldots, F_n$ are KD45-satisfiable iff for each label w in W' the set of $\mathcal{ALC}$-formulas with label w in W' is satisfiable.

Termination of the frame algorithm is stated by the next proposition. Its proof will employ techniques which have been developed for proving termination of term rewriting systems (see [11]).

Proposition 3.3 *If W is a world constraint system which is induced by a finite set of $\mathcal{ALC_B}$-formulas, there is no infinite chain of applications of propagation rules to W.*

In order to prove this proposition we will map derived systems to multisets which are equipped with a well-founded strict partial ordering $\gg$. Multisets are like sets, but multiple occurrences of elements are allowed, e.g., $\{2,3,3,4\}$ is a multiset which is different from the set $\{2,3,4\}$.

A given ordering $>$ on elements in a set S can be extended to an ordering $\gg$ on finite multisets over S as follows. If M and M' are finite multisets over S then $M \gg M'$ iff M' is obtained from M by replacing one or more elements in M by a finite number of elements in S, each of which is smaller than one of the replaced elements (w.r.t. $>$). For example, $\{2,3,3,4\}$ is larger than the multisets $\{2,3,1,2,3\}$ and $\{4\}$. Dershowitz and Manna [11] showed that the ordering $\gg$ on finite multisets over S is well-founded if the original ordering on S is so.

We will use a mapping Ψ which maps derived systems to multisets whose elements are pairs of non-negative integers. These pairs are ordered lexicographically from left to right, i.e., (c_1, c_2) is larger than (c_1', c_2') iff (*i*) $c_1 > c_1'$ or (*ii*) $c_1 = c_1'$ and $c_2 > c_2'$. Since the orderings on both components are well-founded, the lexicographical ordering on these pairs is also well-founded. Finite multisets of these pairs are now compared w.r.t. the multiset ordering which is induced by this lexicographical ordering. This is the well-founded ordering $\gg$ mentioned above.

In order to simplify notation of the mapping Ψ we introduce the following notions. If $F \,||\, w$ and $G \,||\, w$ are labeled $\mathcal{ALC_B}$-axioms with the same label w, we say that $G \,||\, w$ is a *modal subformula* of $F \,||\, w$ iff there is a (possibly empty) sequence $\circ^*$ of modalities such that $\circ^* G$ and F are identical. For example, $\Diamond_j \Box_i \Diamond_j F \,||\, w$, $\Box_i \Diamond_j F \,||\, w$, $\Diamond_j F \,||\, w$, and $F \,||\, w$ are all modal subformulas of $\Diamond_j \Box_i \Diamond_j F \,||\, w$. If $F \,||\, w$ is a labeled $\mathcal{ALC_B}$-formula we denote the set $\{G \mid G \,||\, w \text{ is a modal subformula of } F \,||\, w\}$ by *MSub* $(F \,||\, w)$. For a derived system W, $MSub^W(w)$ denotes the set $\bigcup_{F \,||\, w \in W} MSub\,(F \,||\, w)$.

Now we can define the mapping Ψ as follows.

Definition 3.4 *Let W_0 be a world constraint system which is induced by a finite number of $\mathcal{ALC_B}$-formulas, let W be a world constraint system which is derived from W_0 by applications of the propagation rules of the $\mathcal{ALC_B}$ frame algorithm, and let w be a label in W. We define*

1. *$\mathcal{S}$ is the power set of $\{MSub^{W_0}(w_0) \mid w_0$ is the (only) label in $W_0\}$, and $N_\mathcal{S}$ is the number of elements in $\mathcal{S}$.*

2. N_L^W *is the number of labels in* W *which are different from* w_0.

3. $\psi_1^W(w)$ *is the number of modalities in the set* $\{F \mid\mid w \mid F \mid\mid w \in W\}$.

4. $\psi_2^W(w)$ *is the cardinality of* $\{w \bowtie_j v \mid w \bowtie_j v \in W$ *for some agent* j *and label* $v\}$.

5. *The mapping* $\psi^W(w)$ *is now defined by the pair*

$$\left(N_S - N_L^W, \psi_1^W(w) - \psi_2^W(w)\right)$$

of integers, and $\Psi(W)$ *is the multiset* $\{\psi^W(w) \mid w$ *is a label in* $W\}$.

Given a derived system W and a label w in W we firstly show that both components of $\psi^W(w)$ are non-negative integers. It is easy to verify that each of the four propagation rules only introduces labeled $\mathcal{ALC}_\mathcal{B}$-formulas $H_1 \mid\mid v, \ldots, H_n \mid\mid v$ such that each H_i is an $\mathcal{ALC}_\mathcal{B}$-formula in $MSub^{W_0}(w_0)$. In other words, the set $\{H_1, \ldots, H_n\}$ is an element in $\mathcal{S}$. Because of the definition of the propagation rules a new label v—together with some labeled $\mathcal{ALC}_\mathcal{B}$-formulas $H_1 \mid\mid v, \ldots, H_n \mid\mid v$—is only introduced if there is no label u in W such that the world constraints $H_1 \mid\mid u, \ldots, H_n \mid\mid u$ are exactly the labeled $\mathcal{ALC}_\mathcal{B}$-formulas with label u in W. That means, for each element in $\mathcal{S}$ at most one new label can be introduced, and hence $N_S - N_L^W \geq 0$ for each derived system W. On the other hand, since each propagation rule can be applied to a modality in a labeled $\mathcal{ALC}_\mathcal{B}$-axiom at most once (because of the disjoint preconditions of the four propagation rules) and adds exactly one world constraint of the form $w \bowtie_i v$, we can conclude that $\psi_1^W(w) - \psi_2^W(w) \geq 0$ for each label w in W.

We will now show that each chain $W_0 \rightarrow_1 W_1 \rightarrow_2 \ldots \rightarrow_n W_n$ of propagation rule applications to derived systems corresponds to the decreasing chain $\Psi(W_0) \gg \Psi(W_1) \gg \ldots \gg \Psi(W_n)$. Thus, proposition 3.3 is an immediate consequence of the next lemma.

Lemma 3.5 *If* W' *is obtained from the derived system* W *by an application of a propagation rule, then* $\Psi(W) \gg \Psi(W')$.

Proof: We have to show this lemma for each of the propagation rules.

(1) Assume that W' is obtained from W by applying the $\rightarrow_\Diamond$ or the $\rightarrow_\Box$ rule to one or more world constraints labeled with label w. Such a rule application adds a world constraint $w \bowtie_j v$ to W, where j is an agent and v is a new label. Hence, the number of labels in W' is greater than the number of labels in W and for each label u in W' the first component of $\psi^{W'}(u)$ is less than the first component of $\psi^W(w)$, i.e., $\Psi(W) \gg \Psi(W')$.

(2) Now assume that W' is obtained from W by an application of the $\rightarrow_{\Diamond_\circ}$ or the $\rightarrow_{\Box_\circ}$ rule. Such a propagation rule application does not introduce a new label

to W. Thus, the first components of $\Psi^W(u)$ and $\Psi^{W'}(u)$ are identical for each label u. Let us now consider how the values $\psi_1^W(w)$ and $\psi_2^W(w)$ are changed. Obviously, $\psi_1^W(w) = \psi_1^{W'}(w)$ and $\psi_2^W(w) = \psi_2^{W'}(w) + 1$. From this it follows immediately that $\Psi^W(w) \gg \Psi^{W'}(w)$. □

Thus, the application of the frame algorithm to a world constraint system W induced by a finite set of $\mathcal{ALC}_\mathcal{B}$-formulas $F_1, \ldots, F_n$ terminates and results a world constraint system, say W'. In order to test KD45-satisfiability of W', for each label w in W' we determine the set of all those $\mathcal{ALC}_\mathcal{B}$-formulas which are labeled by w and which do not contain any modality. That means, such a set contains only $\mathcal{ALC}$-formulas and is therefore called the *$\mathcal{ALC}$ test set of label w*. More formally, if W' is a world constraint system, the $\mathcal{ALC}$ test set $A(w)$ of label w in W' is given by the set

$$\{F \mid F \,||\, w \in W' \text{ and } F \text{ does not contain any modality}\}.$$

We will show in the following that a finite set $F_1, \ldots, F_n$ of $\mathcal{ALC}_\mathcal{B}$-formulas is KD45-satisfiable iff the $\mathcal{ALC}$ test set $A(w)$ of each label in W' is satisfiable. Thereby, W' is the result of the frame algorithm with input $\{F_1 \,||\, w_0, \ldots, F_n \,||\, w_0\}$. One direction is given by the next lemma.

Lemma 3.6 *Let W be a world constraint system which is induced by the finite set $\{F_1, \ldots, F_n\}$ of $\mathcal{ALC}_\mathcal{B}$-formulas, and let W' be the result of the frame algorithm with input W. If $K = (\mathcal{W}, \Gamma, K_I)$ is a KD45 Kripke structure which satisfies W, then for each label w in W' there is a world $w^K \in \mathcal{W}$ such that $K, w^K \models F$ for each labeled $\mathcal{ALC}_\mathcal{B}$-formula $F \,||\, w$ in W'.*

Proof: Since W' is the result of the frame algorithm with input W there is a chain $W = W_0 \rightarrow_1 W_1 \rightarrow_2 \ldots \rightarrow_k W_k = W'$ with $\rightarrow_i \in \{\rightarrow_\diamond, \rightarrow_{\diamond_\circ}, \rightarrow_\Box, \rightarrow_{\Box_\circ}\}$ for $i \in \{1, \ldots, k\}$. We will show that K satisfies each labeled $\mathcal{ALC}_\mathcal{B}$-formula in W' by induction over the number of rule applications in this chain. By assumption, $K = (\mathcal{W}, \Gamma, K_I)$ satisfies $W_0 = \{F_1 \,||\, w_0, \ldots, F_n \,||\, w_0\}$, i.e., there is a world w_0^K in $\mathcal{W}$ such that $K, w_0^K \models F_1, \ldots, K, w_0^K \models F_n$.

We thus can assume that, after j rule applications, for each label w in W_j there is a world w^K in $\mathcal{W}$ such that $K, w^K \models F$ for each labeled $\mathcal{ALC}_\mathcal{B}$-formula $F \,||\, w$ in W_j. If $W_j \rightarrow_j W_{j+1}$ there are four possibilities. Firstly, suppose $W_j \rightarrow_\diamond W_{j+1}$ by applying the $\rightarrow_\diamond$ rule to $\diamond_i F \,||\, w$ in W_j. In this case, there are labeled $\mathcal{ALC}_\mathcal{B}$-formulas $\diamond_i F \,||\, w, \diamond_i F_1 \,||\, w, \ldots, \diamond_i F_n \,||\, w, \Box_i G_1 \,||\, w, \ldots, \Box_i G_m \,||\, w$ in W_j, and W_{j+1} is given by

$$W_j \cup \{w \bowtie_i v, F \,||\, v, \diamond_i F_1 \,||\, v, \ldots, \diamond_i F_n \,||\, v, \Box_i G_1 \,||\, v, \ldots, \Box_i G_m \,||\, v, G_1 \,||\, v, \ldots, G_m \,||\, v\}$$

where v is a new label. By induction hypothesis there is a world w^K in $\mathcal{W}$ such that (i) $K, w^K \models \diamond_i F$ and (ii) $K, w^K \models \Box_i G_j$ for $j = 1, \ldots, m$. Because of (i) there is a world v^K in $\mathcal{W}$ (not necessarily different from w^K) such that $(w^K, v^K) \in \gamma_i$ and

$K, v^K \models F$. Furthermore, because of (*ii*) and property (P2) of KD45 Kripke structures, both $K, w^K \models \Box_i G_j$ and $K, w^K \models \Box_i \Box_i G_j$ holds for $j = 1, \ldots, m$. And thus, since $(w^K, v^K) \in \gamma_i$, especially $K, v^K \models G_j$ and $K, v^K \models \Box_i G_j$ holds for $j = 1, \ldots, m$. Finally, $K, w^K \models \Diamond_i F_j$ for $j = 1, \ldots, n$ by induction hypotheses, i.e., $K, w^K \models \Box_i \Diamond_i F_j$ because of property (P3′) of KD45 Kripke structures. Hence, also $K, v^K \models \Diamond_i F_j$ holds for $j = 1, \ldots, n$. Summing up, for each labeled $\mathcal{ALC}_\mathcal{B}$-formula $F \,||\, v$ in W_j we have $K, v^K \models F$.

Secondly, if $W_j \rightarrow_\Box W_{j+1}$ there are world constraints $\Box_i G_1 \,||\, w, \ldots, \Box_i G_m \,||\, w$ in W_j and $W_{j+1} = W_j \cup \{w \bowtie_i v, G_1 \,||\, v, \Box_i G_1 \,||\, v, \ldots, G_m \,||\, v, \Box_i G_m \,||\, v\}$ where v is a new label. By induction hypothesis, $K, w^K \models \Box_i G_j$ for $j = 1, \ldots, m$ and for some world $w^K \in \mathcal{W}$. Because of $K, w^K \models \Box_i G_1$ and property (P1′) of KD45 Kripke structures especially $K, w^K \models \Diamond_i G_1$ holds. Hence, there is a world v^K (not necessarily different from w^K) such that $(w^K, v^K) \in \gamma_i$ and $K, v^K \models G_1$. Furthermore, because of property (P2) of KD45 Kripke structures, both $K, w^K \models \Box_i G_j$ and $K, w^K \models \Box_i \Box_i G_j$ holds for $j = 1, \ldots, m$. And thus, since $(w^K, v^K) \in \gamma_i$, especially $K, v^K \models G_j$ and $K, v^K \models \Box_i G_j$ holds for $j = 1, \ldots, m$. Summing up, we have $K, v^K \models \Box_i G_j$ and $K, v^K \models G_j$ for $j = 1, \ldots, m$.

Finally, if $W_j \rightarrow_{\Diamond_\circ} W_{j+1}$ or $W_j \rightarrow_{\Box_\circ} W_{j+1}$ there is nothing to show since these rules do not change the set of labeled $\mathcal{ALC}_\mathcal{B}$-formulas in W_j at all. □

The next lemma states that a world constraint system W', which is a result of the frame algorithm, is KD45-satisfiable if the $\mathcal{ALC}$ test set of each label in W' is satisfiable.

Lemma 3.7 *Let W be a world constraint system which is induced by a finite set of $\mathcal{ALC}_\mathcal{B}$-formulas, and let W' be the result of the frame algorithm with input W. If the $\mathcal{ALC}$ test set $A(w)$ of each label w in W' is satisfiable, then W is KD45-satisfiable.*

Proof: Let K be the Kripke structure $(\mathcal{W}, \Gamma, K_I)$ where

- $\mathcal{W}$ is given by the set of all labels in W'.
- Γ consists of an accessibility relation γ_i for each agent i. Thereby, γ_i is given by the Euclidean and transitive closure of the set $\{(w, v) \mid w \bowtie_i v \text{ in } W'\} \cup \{(w, w) \mid$ w is a label in W' and for no label v in W' there is $w \bowtie_i v$ in $W'\}$. It is easy to verify that each accessibility relation γ_i is serial.
- K_I is given such that $K, w \models F$ for each labeled $\mathcal{ALC}_\mathcal{B}$-formula $F \,||\, w$ in W' where F does not contain any modality. Such a K-interpretation K_I exists, since we assumed the $\mathcal{ALC}$ test set of each label in W' to be satisfiable. Given interpretations $I_1, \ldots, I_n$ which satisfy the $\mathcal{ALC}$ test sets of each label in W' respectively, the construction of K_I is straightforward.

Obviously, each γ_i is Euclidean, transitive, and serial and hence K is a KD45 Kripke structure.

We will now prove that K satisfies each world constraint c in W'. If c is of the form $w \bowtie_i v$ there is nothing to show because of the definition of K. The fact $K \models F \,||\, w$ for each labeled $\mathcal{ALC}_\mathcal{B}$-formula $F \,||\, w$ in W' can be shown by induction over the number of modalities in F. If F does not contain any modality, then $K, w \models F$ because of the construction of K. Thus we can assume that $K, w \models F$ for each labeled $\mathcal{ALC}_\mathcal{B}$-formula $F \,||\, w$ in W' such that F contains n modalities.

If F contains $n+1$ modalities, there are two possibilities: the leading modality is either $\Box_i$ or $\Diamond_i$. Firstly, suppose W' contains a world constraint $\Box_i F \,||\, w$, where F has n modalities. We then have to show that $K, w \models \Box_i F$, i.e., that $K, v \models F$ for each v such that $(w, v) \in \gamma_i$. Since $\Box_i F \,||\, w$ is in W', during the frame algorithm a propagation rule has been applied to the world constraints with label w, such that $w \bowtie_i u$ is in W' for some label u. It is easy to verify that $w \bowtie_i v \in W'$ for some label v (not necessarily different from w) if $\Box_i F \,||\, w$ or $\Diamond_i F \,||\, w$ is in W' for some $\mathcal{ALC}_\mathcal{B}$-formula F. Thus, because each γ_i is transitive and Euclidean, there are two possibilities for $(w, v) \in \gamma_i$:

1. there is a path $w = w_{i_1} \bowtie_i w_{i_2}, w_{i_2} \bowtie_i w_{i_3}, \ldots, w_{i_{k-1}} \bowtie_i w_{i_k} = v$ in W', or

2. there are two paths starting with some label z, namely

$$z = w_{i_1} \bowtie_i w_{i_2}, w_{i_2} \bowtie_i w_{i_3}, \ldots, w_{i_{k-1}} \bowtie_i w_{i_k} = v \text{ and}$$
$$z = \tilde{w}_{j_1} \bowtie_i \tilde{w}_{j_2}, \tilde{w}_{j_2} \bowtie_i \tilde{w}_{j_3}, \ldots, \tilde{w}_{j_{l-1}} \bowtie_i \tilde{w}_{j_l} = w \text{ in } W'.$$

For case 1., assume that W' contains a world constraint $\Box_i F \,||\, w$. Because of the definition of the propagation rules it holds that, whenever $\Box_i F \,||\, w$ is in W' and $w \bowtie_i v$ (or $w \bowtie_i w$) is added to some derived system W_j, then both $F \,||\, v$ and $\Box_i F \,||\, v$ are in W_j. Analogously, whenever a world constraint $v \bowtie_i u$ is added to a system $W_{j'}$ (with $j' \geq j+1$), the derived system $W_{j'}$ contains $\Box_i F \,||\, u$ and $F \,||\, u$, and so on. Hence, since none of the propagation rules deletes a world constraint, we can conclude that $F \,||\, v$ is in W' if there is a path $w = w_{i_1} \bowtie_i w_{i_2}, w_{i_2} \bowtie_i w_{i_3}, \ldots, w_{i_{k-1}} \bowtie_i w_{i_k} = v$ in W'. And, by induction hypothesis, we know that $K, v \models F$ because F contains only n modalities.

For the second case, assume the two above given paths starting with label z are in W'. We firstly show that

$$(\star) \qquad \text{if } \Box_i F \,||\, w \text{ is in } W', \text{ then } \Box_i F \,||\, z \text{ is also in } W'.$$

It is sufficient to show that $\Box_i F \,||\, u$ is in W_j whenever $\Box_i F \,||\, u'$ is in W_{j+1}, where $W_j \rightarrow_\Diamond W_{j+1}$, $W_j \rightarrow_{\Diamond_\circ} W_{j+1}$, $W_j \rightarrow_\Box W_{j+1}$, or $W_j \rightarrow_{\Box_\circ} W_{j+1}$, and $u \bowtie_i u'$ is added to W_j by this rule application. This holds because of the definitions of the four propagation rules and since $\mathcal{ALC}_\mathcal{B}$-formulas are in KD45 normal form, such that $(\star)$ follows immediately.

1. Let W be the world constraint system which is induced by $F_1, \ldots, F_n$.
2. Let W' be the result of the frame algorithm with input W.
3. For each label w in W' do: If the $\mathcal{ALC}$ test set of w is not satisfiable, then STOP and return "KD45-unsatisfiable".
4. Return "KD45-satisfiable".

Figure 2: The KD45-satisfiability algorithm.

That means, if $\Box_i F \,||\, w$ is in W' we know that $\Box_i F \,||\, z$ is in W'. As argued above, in this case W' contains the labeled $\mathcal{ALC}_\mathcal{B}$-formula $F \,||\, v$ because of the path $z \bowtie_i \ldots \bowtie_i v$. Again, we know $K, v \models F$ because of the induction hypothesis.

Secondly suppose W' contains $\Diamond_i F \,||\, w$. We then have to show that $K, v \models F$ for some world v such that $(w, v) \in \gamma_i$. This is obvious, since either (i) the $\rightarrow_\Diamond$ rule has been applied to $\Diamond_i F \,||\, w$ and added both world constraints $w \bowtie_i v$ and $F \,||\, v$ for some label v, or (ii) the $\rightarrow_{\Diamond_0}$ rule has been applied to $\Diamond_i F \,||\, w$ and added $w \bowtie_i u$ to W' such that $F \,||\, u \in W'$. Summing up, K satisfies each world constraint in W' and hence in $W \subseteq W'$. Thus, W is KD45-satisfiable. □

The next theorem summarizes the previous results.

Theorem 3.8 *Let $F_1, \ldots, F_n$ be a finite set of $\mathcal{ALC}_\mathcal{B}$-formulas, and let W be the world constraint system which is induced by $F_1, \ldots, F_n$. If W' is the result of the frame algorithm with input W, then the set $F_1, \ldots, F_n$ is KD45-satisfiable iff the $\mathcal{ALC}$ test set $A(w)$ of each label w in W' is satisfiable.*

Proof: The $\mathcal{ALC}_\mathcal{B}$-formulas $F_1, \ldots, F_n$ are KD45-satisfiable iff W is KD45-satisfiable. Firstly, suppose K is a KD45 Kripke structure which satisfies W. Then, because of Lemma 3.6, for each label w in W' there is a world $w^K \in \mathcal{W}$ such that $K, w^K \models F$ for each $\mathcal{ALC}_\mathcal{B}$-formula $F \,||\, w$ in W'. Thus, especially the $\mathcal{ALC}$ test set of each label w in W' is satisfied by K. Conversely, suppose that the $\mathcal{ALC}$ test set $A(w)$ of each label w in W' is satisfiable. Then W is KD45-satisfiable because of Lemma 3.7. □

Summing up, we obtain the algorithm for testing KD45-satisfiability of $\mathcal{ALC}_\mathcal{B}$-formulas $F_1, \ldots, F_n$ which is given in Figure 2. An algorithm for testing satisfiability of $\mathcal{ALC}$ test sets has been given in [21].

Unfortunately, the number of labels in the constructed world constraint system is in the worst case exponential in the number of input $\mathcal{ALC}_\mathcal{B}$-formulas: Let $\Diamond_1 F_1, \ldots, \Diamond_1 F_n$ be $\mathcal{ALC}_\mathcal{B}$-formulas where each F_i is an $\mathcal{ALC}$-formula. The induced world constraint system W_0 is then given by

$$W_0 = \{\Diamond_1 F_1 \mid\mid w_0, \ldots, \Diamond_1 F_n \mid\mid w_0\}.$$

Applying the $\rightarrow_\Diamond$ rule successively to $\Diamond_1 F_1 \mid\mid w_0, \ldots, \Diamond_1 F_n \mid\mid w_0$ results the world constraint systems

$$\begin{array}{rcl} W_1 & = & W_0 \cup \{w_0 \bowtie_1 w_1, F_1 \mid\mid w_1, \Diamond_1 F_2 \mid\mid w_1, \ldots, \Diamond_1 F_n \mid\mid w_1\} \\ W_2 & = & W_1 \cup \{w_0 \bowtie_1 w_2, \Diamond_1 F_1 \mid\mid w_2, F_2 \mid\mid w_2, \Diamond_1 F_3 \mid\mid w_2, \ldots, \Diamond_1 F_n \mid\mid w_2\} \\ & \vdots & \\ W_n & = & W_{n-1} \cup \{w_0 \bowtie_1 w_n, \Diamond_1 F_1 \mid\mid w_n, \ldots, \Diamond_1 F_{n-1} \mid\mid w_n, F_n \mid\mid w_n\} \end{array}$$

Each of the world constraint systems $W_1, \ldots, W_n$ contains $n-1$ labeled $\mathcal{ALC}_\mathcal{B}$-formulas with a leading modality $\Diamond_1$, i.e., to each of these derived systems the $\rightarrow_\Diamond$ rule can be applied $n-1$ times.[4] To each of the thereby obtained derived systems the $\rightarrow_\Diamond$ rule can be applied $n-2$ times and so on. summing up, an exponential number of labels—and hence of labeled $\mathcal{ALC}_\mathcal{B}$-formulas—is generated.

In order to test KD45-satisfiability of a set of $\mathcal{ALC}_\mathcal{B}$-formulas we are mainly interested in the constructed $\mathcal{ALC}$ test sets which have to be tested on satisfiability. The following example shows that the number of different $\mathcal{ALC}$ test sets in a derived system W may essentially be smaller than the number of labels in W.

Example 3.9 *Let the $\mathcal{ALC}_\mathcal{B}$-formulas $\Diamond_1 F_1, \Diamond_1 F_2, \Diamond_1 F_3$ be given where F_1, F_2, F_3 are $\mathcal{ALC}$-formulas. Applying the frame algorithm to*

$$\{\Diamond_1 F_1 \mid\mid w_0, \Diamond_1 F_2 \mid\mid w_0, \Diamond_1 F_3 \mid\mid w_0\}$$

results a world constraint system W' which has 13 different labels. However, only three different $\mathcal{ALC}$ test sets (namely $\{F_1\}$, $\{F_2\}$, and $\{F_3\}$) are constructed.

An optimized version of the KD45-satisfiability algorithm—which does not generate a worst case exponential overhead of labeled $\mathcal{ALC}_\mathcal{B}$-formulas—is presented in the following section.

4 Optimization and Computing $\mathcal{ALC}_\mathcal{B}$-Inferences

In this section we consider optimizations of the KD45-satisfiability algorithm as well as the problem of computing logical inferences from given $\mathcal{ALC}_\mathcal{B}$-formulas. In 4.1 we

[4]For sake of simplicity we disregard the $\rightarrow_\circ$ rule here. It is easy to verify that applications of this rule do not influence the exponential behaviour of the frame algorithm with input $\Diamond_1 F_1, \ldots, \Diamond_1 F_n$.

will present an algorithm which—based upon the results of the previous section—determines $\mathcal{ALC}$ test sets without computing an exponential number of labeled $\mathcal{ALC}_{\mathcal{B}}$-formulas. These sets then have to be tested on satisfiability, i.e., we have to decide whether or not a set of terminological and assertional axioms is satisfiable. Terminological axioms are defined to be of the form $C = D$ or $C \neq D$ where C, D are (complex) concepts. However, in most of the existing terminological representation systems terminological axioms are not allowed in this general form but they have to satisfy additional conditions. In 4.2 we investigate in which cases only such restricted terminological axioms have to be considered in order to test satisfiability of $\mathcal{ALC}$ test sets. Finally, in 4.3 we will show how to decide whether or not a given $\mathcal{ALC}_{\mathcal{B}}$-formula is logically entailed by a set of $\mathcal{ALC}_{\mathcal{B}}$-formulas.

4.1 Computing $\mathcal{ALC}$ test sets

The KD45-satisfiability algorithm presented in the previous section works in two phases: Firstly, a set of $\mathcal{ALC}$ test sets is computed and then each of these sets is tested on satisfiability. The thereby used propagation rules of the frame algorithm have the advantage to "reflect" the properties of KD45 Kripke structures, and hence soundness and completeness of the KD45-satisfiability algorithm could be proved in a very natural way. On the other hand, the frame algorithm has the disadvantage to possibly construct a large number of labeled $\mathcal{ALC}_{\mathcal{B}}$-formulas in order to determine a small number of $\mathcal{ALC}$ test sets (cf. example 3.9).

In the following we will have a closer look at the properties of the frame algorithm. Building upon these properties we will then develop an algorithm which computes $\mathcal{ALC}$ test sets immediately from the $\mathcal{ALC}_{\mathcal{B}}$-formulas to be tested on KD45-satisfiability, i.e., we do no longer have to compute a (large) number of labels from which $\mathcal{ALC}$ test sets then are extracted.

The main idea of this new approach is as follows: Suppose $\circ_1^* F_1, \ldots, \circ_n^* F_n$ are $\mathcal{ALC}_{\mathcal{B}}$-formulas which are to be tested on KD45-satisfiability, where each $\circ_i^*$ is a (possibly empty) sequence of modalities and each F_i is an $\mathcal{ALC}$-formula. By looking at the structure of the sequences $\circ_1^*, \ldots, \circ_n^*$ we will then decide syntactically which elements in the power set of $\{F_1, \ldots, F_n\}$ will be computed as an $\mathcal{ALC}$ test set by the frame algorithm with input $\circ_1^* F_1 \,||\, w_0, \ldots, \circ_n^* F_n \,||\, w_0$. The following example shows that this task is not obvious.

Example 4.1 Let A be a primitive concept and let C, D be concepts.

1. Let $\mathcal{S}$ be the set $\{\Diamond_1(A = C), \Diamond_1(A = D)\}$. Applying the frame algorithm to the induced world constraint system $W = \{\Diamond_1(A = C) \,||\, w_0, \Diamond_1(A = D) \,||\, w_0\}$

results the derived system W', given by the world constraints

$$\begin{array}{lll} & \Diamond_1(A=C) \parallel w_0 & \Diamond_1(A=D) \parallel w_0 \\ w_0 \bowtie_1 w_1 & A=C \parallel w_1 & \Diamond_1(A=D) \parallel w_1 \\ w_0 \bowtie_1 w_2 & \Diamond_1(A=C) \parallel w_2 & A=D \parallel w_2 \\ w_1 \bowtie_1 w_3 & & A=D \parallel w_3 \\ w_2 \bowtie_1 w_4 & A=C \parallel w_4 & \end{array}$$

The $\mathcal{ALC}$ test set $A(w_0)$ of w_0 is empty, while $A(w_1) = A(w_4) = \{A = C\}$ and $A(w_2) = A(w_3) = \{A = D\}$.

2. On the other hand, starting with the set $\mathcal{S} = \{\Box_1(A = C), \Box_1(A = D)\}$ leads to only one non-empty $\mathcal{ALC}$ test set, namely $\{A = C, A = D\}$.

The leading sequences of modalities in the input $\mathcal{ALC_B}$-formulas obviously influence the determined $\mathcal{ALC}$ test sets. ∎

Let us firstly introduce the following notions. If W is a derived system, a subset consisting of world constraints $w_1 \bowtie_{i_1} w_2, \ldots, w_{n-1} \bowtie_{i_{n-1}} w_n$ is called a *path* if the w_j are labels in W and each i_j is an agent. The *norm* of this path is given by the sequence $i_{1'} \ldots i_{n'}$ which arises from $i_1 \ldots i_{n-1}$ be replacing each subsequence $i_j \ldots i_j$ in $i_1 \ldots i_{n-1}$ by i_j. For example, the norm of the path $w_1 \bowtie_{i_1} w_2, w_2 \bowtie_{i_1} w_3, w_3 \bowtie_{i_2} w_4, w_4 \bowtie_{i_2} w_5$ is given by $i_1 i_2$.

In the following we will present correlations between labeled $\mathcal{ALC_B}$-formulas which are taken as input of the frame algorithm and the labeled $\mathcal{ALC_B}$-formulas which are generated during the frame algorithm with this input. In order to simplify notation we thereby assume the input is of the form $\circ_1^* F_1 \parallel w_0, \ldots, \circ_n^* F_n \parallel w_0$ where each $\circ_i^*$ is a (possibly empty) sequence of modalities and each two $\mathcal{ALC}$-formulas F_i and F_j are syntactically different (e.g., by marking them with numbers). Since the presented correlations only depend on the modalities in the input $\mathcal{ALC_B}$-formulas this can be done without loss of generality.

Firstly, if W' is the result of an application of the frame algorithm, the next lemma states that there is a correlation between paths in W' and the sets of $\mathcal{ALC_B}$-formulas in W'.

Lemma 4.2 *Let W be a world constraint system which is induced by a finite set of $\mathcal{ALC_B}$-formulas, let W' be the result of the frame algorithm with input W, and let P be a path $w_0 \bowtie_{i_1} w_1, \ldots, w_{n-1} \bowtie_{i_n} w_n$ in W' with norm $(P) = i_1 \ldots i_n$. Then the set of labeled $\mathcal{ALC_B}$-formulas in W' with label w_n is a subset of $\{F \parallel w_n, \circ_{i_n} F \parallel w_n \mid \circ_{i_1} \ldots \circ_{i_n} F \parallel w_0 \in W'\}$ where $\circ \in \{\Box, \Diamond\}$.*

Proof: Let $w_0 \bowtie_i u$ be in W'. If $\Diamond_i F_1 \,||\, w_0, \ldots, \Diamond_i F_n \,||\, w_0, \Box_i G_1 \,||\, w_0, \ldots, \Box_i G_m \,||\, w_0$ are the labeled $\mathcal{ALC}_\mathcal{B}$-formulas in W' with label w_0 and with leading modality $\Box_i$ or $\Diamond_i$, the labeled $\mathcal{ALC}_\mathcal{B}$-formulas in W' with label u are a subset of S, given by

$$\{\Diamond_i F_1 \,||\, u, \ldots, \Diamond_i F_n \,||\, u, \Box_i G_1 \,||\, u, \ldots, \Box_i G_m \,||\, u, F_1 \,||\, u, \ldots, F_n \,||\, u, G_1 \,||\, u, \ldots, G_m \,||\, u\}$$

or, alternatively, by $\{F \,||\, u, \circ_i F \,||\, u \mid \circ_i F \,||\, w_0 \in W'\}$. This follows immediately from the definition of the four propagation rules. Since we assumed $\mathcal{ALC}_\mathcal{B}$-formulas to be in KD45 normal form, none of the world constraints $F_1 \,||\, u, \ldots, F_n \,||\, u, G_1 \,||\, u, \ldots, G_m \,||\, u$ has a leading modality $\Box_i$ or $\Diamond_i$. Hence, if for some label u' there is $u \bowtie_i u'$ in W', the labeled $\mathcal{ALC}_\mathcal{B}$-formulas with label u' in W' also are a subset of S, and so on. Summing up, if there is a path $w_0 \bowtie_i w_{i_1}, \ldots, w_{i_{k-1}} \bowtie_i w_k$ we know that the set of labeled $\mathcal{ALC}_\mathcal{B}$-formulas with label w_k in W' is a subset of $S = \{F \,||\, w_k, \circ_i F \,||\, w_k \mid \circ_i F \,||\, w_0 \in W'\}$.

Let us now consider a path P with *norm* $(P) = ij$. If there is a path $w_0 \bowtie_i w_{i_1}, \ldots, w_{i_{k-1}} \bowtie_i w_k, w_k \bowtie_j w_{j_1}$, then the set of labeled $\mathcal{ALC}_\mathcal{B}$-formulas with label w_k in W' is a subset of $S = \{F \,||\, w_k, \circ_i F \,||\, w_k \mid \circ_i F \,||\, w_0 \in W'\}$. Hence, the set of labeled $\mathcal{ALC}_\mathcal{B}$-formulas with label w_{j_1} in W' is a subset of $\{F \,||\, w_{j_1}, \circ_j F \,||\, w_{j_1} \mid \circ_j F \,||\, w_k \in S\}$. Analogously to the argumentation above we obtain the following: If there is a path $w_0 \bowtie_i w_{i_1}, \ldots, w_{i_{k-1}} \bowtie_i w_k, w_k \bowtie_j w_{j_1}, \ldots, w_{l-1} \bowtie_j w_l$ in W' it follows that the set of labeled $\mathcal{ALC}_\mathcal{B}$-formulas with label w_l in W' is a subset of $\{F||w_l, \circ_j F||w_l \mid \circ_j F||w_k \in S\}$.

Since S consists only of elements $\circ_i F \,||\, w_k$ or $F \,||\, w_k$ such that $\circ_i F \,||\, w_0$ is in W' it follows that $\circ_j F \,||\, w_k \in S$ at most if $\circ_i \circ_j F \,||\, w_0 \in W'$. That means, the set of labeled $\mathcal{ALC}_\mathcal{B}$-formulas with label w_l in W' is a subset of $\{F \,||\, w_l, \circ_j F \,||\, w_l \mid \circ_i \circ_j F \,||\, w_0 \in W'\}$.

With this argumentation the lemma follows immediately by induction. □

In an $\mathcal{ALC}$ test set only $\mathcal{ALC}$-formulas, i.e. $\mathcal{ALC}_\mathcal{B}$-formulas without modalities, do occur. If W is a world constraint system and W' is the result of applying the frame algorithm to W, the next lemma provides a correlation between labeled $\mathcal{ALC}_\mathcal{B}$-formulas in W' without modalities and labeled $\mathcal{ALC}_\mathcal{B}$-formulas in W. Thereby, a sequence of modalities is called *sequence* for short. If S is the sequence $\circ_1 \ldots \circ_n$, with $\circ \in \{\Diamond, \Box\}$, then S$[j]$ denotes $\circ_j$ and *indexes* (S) denotes $1, \ldots, n$. Furthermore, an $\mathcal{ALC}_\mathcal{B}$-formula F *occurs* in a derived system iff there is a (possibly empty) sequence $\circ^*$ and a label w in W such that $\circ^* F \,||\, w$ is in W.

Lemma 4.3 *Let W be a world constraint system which is induced by a finite number of $\mathcal{ALC}_\mathcal{B}$-formulas, let W' be the result of the frame algorithm with input W, and let F and G be $\mathcal{ALC}$-formulas occurring in W. If there is a label w_n in W' such that $F \,||\, w_n$ and $G \,||\, w_n$ are both in W', then there are two sequences* S *and* S′ *such that (i)* S$F \,||\, w_0$ *and* S′$G \,||\, w_0$ *are in W and (ii) indexes* (S) = *indexes* (S′).

Proof: Let $w_0 \bowtie_{i_1} w_1, \ldots, w_{n-1} \bowtie_{i_n} w_n$ be the path from w_0 to w_n in W' where each $w_{j-1} \bowtie_{i_j} w_j$ has been added into W' by an application of the $\rightarrow_\Diamond$ or the $\rightarrow_\Box$ rule. It is sufficient to investigate this path since the $\rightarrow_{\Diamond_\circ}$ and the $\rightarrow_{\Box_\circ}$ rule do not add any new labeled $\mathcal{ALC}_\mathcal{B}$-formulas at all, and it is easy to verify that there is exactly one such path.

If $F || w_n$ and $G || w_n$ are both in W', there are labeled $\mathcal{ALC}_\mathcal{B}$-formulas $\circ_{i_1} \ldots \circ_{i_n} F || w_0$ and $\circ_{i_1} \ldots \circ_{i_n} G || w_0$ in W where each $\circ$ is either $\Diamond$ or $\Box$ and each i_j is an agent. This follows immediately by Lemma 4.2 and the fact that $H || w_0 \in W'$ iff $H || w_0 \in W$ for each $\mathcal{ALC}_\mathcal{B}$-formula H since no propagation rule introduces a labeled $\mathcal{ALC}_\mathcal{B}$-formula with label w_0 into a derived system. Thus, there are sequences S and S' such that *indexes* $(\mathsf{S}) =$ *indexes* (S') and both $\mathsf{S}F || w_0$ and $\mathsf{S}'G || w_0$ are in W. □

If W' is the result of applying the frame algorithm to a world constraint system W, the following proposition provides syntactical conditions that W satisfies whenever there is an $\mathcal{ALC}$ test set $A(w)$ for some label w in W which contains a non-empty set of $\mathcal{ALC}$-axioms.

Proposition 4.4 *Let W be a world constraint system which is induced by a finite number of $\mathcal{ALC}_\mathcal{B}$-formulas, let W' be the result of the frame algorithm with input W, and let F and G be $\mathcal{ALC}$-formulas occurring in W. If there is a label w_n in W' such that $F || w_n$ and $G || w_n$ are both in W', then there are sequences S and S' such that (i) indexes $(\mathsf{S}) =$ indexes (S'), (ii) $\mathsf{S}F || w_0$ and $\mathsf{S}'G || w_0$ are in W, and (iii) there is no position j in S (respectively in S') such that $\mathsf{S}[j] = \mathsf{S}'[j] = \Diamond_{i_j}$ for some agent i_j.*

Proof: Because of Lemma 4.3 we know that there are sequences S and S' with *indexes* $(\mathsf{S}) =$ *indexes* (S') and both $\mathsf{S}F || w_0$ and $\mathsf{S}'G || w_0$ are in W. Suppose j is the first position such that $\mathsf{S}[j] = \mathsf{S}'[j] = \Diamond_{i_j}$.

Starting with label w_0 we firstly show that for each path $w_0 \bowtie_{i_1} w_1, \ldots, w_{n-1} \bowtie_{i_n} w_n$ in W' there is a label w_j in $w_0, \ldots, w_n$ such that $\Diamond_{i_j} \circ_1^* F || w_j$ and $\Diamond_{i_j} \circ_2^* G || w_j$ are in W', where $\circ_1^*$ and $\circ_2^*$ are (possibly empty) sequences of modalities and *indexes* $(\circ_1^*) =$ *indexes* $(\circ_2^*)$. Therefore we use the fact that for each position $k < j$ at least one of the modalities in $\{\mathsf{S}[k], \mathsf{S}'[k]\}$ is not a labeled $\Diamond$-modality.

If $\Diamond_{i_j} \circ_1^* F || w_0$ and $\Diamond_{i_j} \circ_2^* G || w_0$ are both in W' there is nothing to show. Otherwise, we have to distinguish two cases:

(i) $\Diamond_i \circ^* \Diamond_{i_j} \circ_1^* F || w_0$ and $\Box_i \circ^* \Diamond_{i_j} \circ_2^* G || w_0$ are in W' or
(ii) $\Box_i \circ^* \Diamond_{i_j} \circ_1^* F || w_0$ and $\Box_i \circ^* \Diamond_{i_j} \circ_2^* G || w_0$ are in W',

where i is an agent and $\circ^*$ is a (possibly empty) sequence of modalities.[5] Without loss of generality we do not consider the case where $\Box_i \circ^* \Diamond_{i_j} \circ_1^* F || w_0$ and $\Diamond_i \circ^* \Diamond_{i_j} \circ_2^* G || w_0$

[5] In order to simplify notation we will use $\circ^*$ in both the sequences of F and of G. Formally we had to distinguish these occurrences since everything we know about them is that they do not differ in their indexes. However, here it should always be clear by context which occurrence is meant.

are in W', since it is symmetrical to case (i). Note, that applying propagation rules to labeled $\mathcal{ALC}_{\mathcal{B}}$-formulas with label w_0 whose leading modalities are not indexed with i—obtaining, say, label v—cannot result the first element in a path leading to a label w such that $F \,||\, w$ and $G \,||\, w$ are both in W'. This is due to the fact that in this case neither F nor G occurs in labeled $\mathcal{ALC}_{\mathcal{B}}$-formulas labeled with v.

For case (i), after an application of the $\rightarrow_\Diamond$ or the $\rightarrow_{\Diamond_\circ}$ rule to $\Diamond_i \circ^* \Diamond_{i_j} \circ_1^* F \,||\, w_0$, for some label v the world constraints

$$\circ^*\Diamond_{i_j} \circ_1^* F \,||\, v, \quad \circ^*\Diamond_{i_j} \circ_2^* G \,||\, v, \quad \Box_i \circ^* \Diamond_{i_j} \circ_2^* G \,||\, v \qquad (\alpha)$$

are exactly the labeled $\mathcal{ALC}_{\mathcal{B}}$-formulas in the currently derived world constraint system which are labeled with v and contain F or G. Analogously, after an application of the $\rightarrow_\Diamond$ or the $\rightarrow_{\Diamond_\circ}$ rule to $\Diamond_i H \,||\, w_0$—where H is different from $\circ^*\Diamond_{i_j} \circ_1^* F$—for some label v the world constraints

$$\Diamond_i \circ^* \Diamond_{i_j} \circ_1^* F \,||\, v, \quad \circ^*\Diamond_{i_j} \circ_2^* G \,||\, v, \quad \Box_i \circ^* \Diamond_{i_j} \circ_2^* G \,||\, v \qquad (\beta)$$

are exactly the labeled $\mathcal{ALC}_{\mathcal{B}}$-formulas in the currently derived world constraint system which are labeled with v and contain F or G.

For case (ii), after an application of the $\rightarrow_\Box$ or the $\rightarrow_{\Box_\circ}$ rule to labeled $\mathcal{ALC}_{\mathcal{B}}$-formulas with label w_0 whose leading modalities are indexed by i, for some label v the world constraints

$$\Box_i \circ^* \Diamond_{i_j} \circ_1^* F \,||\, v, \quad \circ^*\Diamond_{i_j} \circ_1^* F \,||\, v, \quad \Box_i \circ^* \Diamond_{i_j} \circ_2^* G \,||\, v, \quad \circ^*\Diamond_{i_j} \circ_2^* G \,||\, v \qquad (\gamma)$$

are exactly the labeled $\mathcal{ALC}_{\mathcal{B}}$-formulas in the currently derived world constraint system which are labeled with v and contain F or G. And, finally, after an application of the $\rightarrow_\Diamond$ or the $\rightarrow_{\Diamond_\circ}$ rule to some labeled $\mathcal{ALC}_{\mathcal{B}}$-formula $\Diamond_i H \,||\, w_0$ we obtain the same labeled $\mathcal{ALC}_{\mathcal{B}}$-formulas as in case (γ) above.

Since we assumed $\mathcal{ALC}_{\mathcal{B}}$-formulas to be in KD45 normal form, the leading modality in $\circ^*$ is not indexed with i. Hence, for case (α), propagation rule applications to $\Box_i \circ^* \Diamond_{i_j} \circ_2^* G \,||\, v$ cannot result the first element in a path leading to a label w such that $F \,||\, w$ and $G \,||\, w$ are both in W'. Analogously, for case (β), applying propagation rules to $\circ^*\Diamond_{i_j} \circ_2^* G \,||\, v$ cannot lead to a label w such that $F \,||\, w$ and $G \,||\, w$ are both in W'. For the same reason in case (γ) either rule applications to labeled $\mathcal{ALC}_{\mathcal{B}}$-formulas with leading modality i and label v, or to labeled $\mathcal{ALC}_{\mathcal{B}}$-formulas whose leading modality is the first modality in $\circ^*$ and label v, may lead to a label w such that $F \,||\, w$ and $G \,||\, w$ are both in W'.

Summing up, if v is the first element in a path $w_0 \bowtie_{i_1}, \ldots, \bowtie_{i_n} w_n$, then either

(1) $\circ^*\Diamond_{i_j} \circ_1^* F \,||\, v$ and $\circ^*\Diamond_{i_j} \circ_2^* G \,||\, v$ are in W' or
(2) $\Diamond_i \circ^* \Diamond_{i_j} \circ_1^* F \,||\, v$ and $\Box_i \circ^* \Diamond_{i_j} \circ_2^* G \,||\, v$ are in W' or
(3) $\Box_i \circ^* \Diamond_{i_j} \circ_1^* F \,||\, v$ and $\Box_i \circ^* \Diamond_{i_j} \circ_2^* G \,||\, v$ are in W'

Hence, to v the same argumentation is applicable as to label w_0 above.

Let now w_j be a label in $w_0, \ldots, w_n$ such that $\Diamond_{i_j} \circ_1^* F \,||\, w_j$ and $\Diamond_{i_j} \circ_2^* G \,||\, w_j$ are in W'. Without loss of generality during the frame algorithm the $\rightarrow_{\Diamond}$ (respectively the $\rightarrow_{\Diamond_\circ}$) rule has been applied to $\Diamond_{i_j} \circ_1^* F \,||\, w_j$ before it has been applied to $\Diamond_{i_j} \circ_2^* G \,||\, w_j$. If $\circ_1^*$ (and hence $\circ_2^*$) is the empty sequence, then $F \,||\, v$ and $\Diamond_{i_j} G \,||\, v$ are in W', where $w_j \bowtie_{i_j} v$ has been introduced by this rule application. But $F \,||\, w_n$ cannot be in W' because of the remaining application of the $\rightarrow_{\Diamond}$ rule to $\Diamond_{i_j} \circ^* G \,||\, w_j$. On the other hand, if $\circ_1^*$ (and hence $\circ_2^*$) is not empty, its leading modality is different from i_j (since $\mathcal{ALC}_{\mathcal{B}}$-formulas are in KD45 normal form) and thus there cannot be a label w_n in W' such that $F \,||\, w_n$ and $G \,||\, w_n$ are both in W' because of the remaining application of the $\rightarrow_{\Diamond}$ or the $\rightarrow_{\Diamond_\circ}$ rule to $\Diamond_{i_j} \circ^* G \,||\, w_j$.

In both cases this contradicts the assumption that there is a label w_n in W' such that $F \,||\, w_n$ and $G \,||\, w_n$ are both in W', i.e., there cannot exist a position j such that $\mathsf{S}[j] = \mathsf{S}'[j] = \Diamond_{i_j}$ for some agent i_j. □

Finally, we present a proposition which states that also the opposite direction of proposition 4.4 holds.

Proposition 4.5 *Let W be a world constraint system which is induced by a finite set of $\mathcal{ALC}_{\mathcal{B}}$-formulas, let W' be the result of the frame algorithm with input W, and let F and G be $\mathcal{ALC}$-formulas which occur in W. If $\mathsf{S}F \,||\, w_0$ and $\mathsf{S}'G \,||\, w_0$ are in W, where S, S' are sequences such that indexes (S) = indexes (S') and there is no position j in S (respectively in S') such that $\mathsf{S}[j] = \mathsf{S}'[j] = \Diamond_{i_j}$ for some agent i_j, then there is a label w_n in W' such that $F \,||\, w_n$ and $G \,||\, w_n$ are both in W'.*

Proof: Let S_* and S'_* arise from S and S' by deleting the first modality, respectively. If $\mathsf{S}F \,||\, w_0$ and $\mathsf{S}'G \,||\, w_0$ are in W, there are three cases:

1. $\mathsf{S}[1] = \mathsf{S}'[1] = \Box_{i_1}$ and the $\rightarrow_{\Box}$ or the $\rightarrow_{\Box_\circ}$ rule is applied to $\mathcal{ALC}_{\mathcal{B}}$-formulas of the form $\Box_{i_1} H \,||\, w_0$. In this case $\mathsf{S}F \,||\, w_1$, $\mathsf{S}'G \,||\, w_1$, $\mathsf{S}_*F \,||\, w_1$, and $\mathsf{S}'_*G \,||\, w_1$ are all in W' if $w_0 \bowtie_{i_1} w_1$ is introduced by this rule application.

2. Without loss of generality $\mathsf{S}[1] = \Diamond_{i_1}$, $\mathsf{S}'[1] = \Box_{i_1}$, and the $\rightarrow_{\Diamond}$ or the $\rightarrow_{\Diamond_\circ}$ rule is applied to some $\mathcal{ALC}_{\mathcal{B}}$-formula $\Diamond_{i_1} H \,||\, w_0$, where H is different from F. In this case $\mathsf{S}F \,||\, w_1$ and $\mathsf{S}'G \,||\, w_1$ are in W' if $w_0 \bowtie_{i_1} w_1$ is introduced by this rule application.

3. Without loss of generality $\mathsf{S}[1] = \Diamond_{i_1}$, $\mathsf{S}'[1] = \Box_{i_1}$, and the $\rightarrow_{\Diamond}$ or the $\rightarrow_{\Diamond_\circ}$ rule is applied to $\mathsf{S}F \,||\, w_0$. In this case $\mathsf{S}_*F \,||\, w_1$ and $\mathsf{S}'_*G \,||\, w_1$ are both in W' if $w_0 \bowtie_{i_1} w_1$ is introduced by this rule application. It does not influence the argumentation that $\mathsf{S}'G \,||\, w_1$ is also in W'.

For label w_1 the argumentation is the same such that the proposition follows immediately by induction. □

Propositions 4.4 and 4.5 together give us an answer to the question which $\mathcal{ALC}$ test sets are generated by an application of the frame algorithm. We summarize this result in the following theorem.

Theorem 4.6 *f Let W be a world constraint system which is induced by a finite set of $\mathcal{ALC}_{\mathcal{B}}$-formulas and let W' be the result of the frame algorithm with input W. Then there is a label w in W' such that the $\mathcal{ALC}$ test set $A(w)$ contains the $\mathcal{ALC}$-formulas $F_1, \ldots, F_n$ iff there are sequences $\mathsf{S}_1, \ldots, \mathsf{S}_n$ in W such that*

- $\mathsf{S}_1 F_1 \,||\, w_0, \ldots, \mathsf{S}_n F_n \,||\, w_0$ *are in W,*
- *indexes*$(\mathsf{S}_1) = \ldots =$ *indexes*(S_n), *and*
- *there is no position j such that for two sequences S' and S'' in $\{\mathsf{S}_1, \ldots, \mathsf{S}_n\}$ holds $\mathsf{S}'[j] = \mathsf{S}''[j] = \Diamond_{i_j}$ for some agent i_j.*

Proof: If the $\mathcal{ALC}$ test set $A(w)$ contains the $\mathcal{ALC}$-axioms $F_1, \ldots, F_n$, none of the F_j does contain modalities. Thus, because of Proposition 4.4 there are sequences $\mathsf{S}_1, \ldots, \mathsf{S}_n$ such that $\mathsf{S}_1 F_1 \,||\, w_0, \ldots, \mathsf{S}_n F_n \,||\, w_0$ are in W and *indexes*$(\mathsf{S}_1) = \ldots =$ *indexes*(S_n). Now suppose, that $\mathsf{S}'F' \,||\, w_0$ and $\mathsf{S}''F'' \,||\, w_0$ with F' and F'' in $\{F_1, \ldots, F_n\}$ are in W such that $\mathsf{S}'[j] = \mathsf{S}''[j] = \Diamond_{i_j}$ for some agent i_j. Again because of Proposition 4.4, we can conclude that either $F' \,||\, w$ or $F'' \,||\, w$ is not in W'. This contradicts that $A(w)$ contains $F_1, \ldots, F_n$. The other direction follows immediately from Proposition 4.5. □

An optimized version of the KD45-satisfiability algorithm is given in Figure 3. For the input of this algorithm remember that each $\mathcal{ALC}_{\mathcal{B}}$-formula is of the form $\mathsf{S}F$ where S is a (possibly empty) sequence of modalities and F is an $\mathcal{ALC}$-formula. In step 2. of the algorithm for each sequence S_i in $\{\mathsf{S}_1, \ldots, \mathsf{S}_n\}$ the set S_i of all $\mathcal{ALC}$-formulas in $\{F_1, \ldots, F_n\}$ is computed such that the conditions of Theorem 4.6 are satisfied. Multiple generations of the same $\mathcal{ALC}$ test sets are avoided by testing whether or not the $\mathcal{ALC}$-formula F_i is already in an $\mathcal{ALC}$ test set S_k.

Summing up, soundness, completeness, and termination of this algorithm follow immediately from the results in Section 3 and in this subsection. It is easy to verify that step 2. of this optimized KD45-satisfiability algorithm works in polynomial time in the length of the input $\mathcal{ALC}_{\mathcal{B}}$-formulas and generates a polynomial number of $\mathcal{ALC}$ test sets. Thus, in contrast to the KD45-satisfiability algorithm in the previous section, it does not generate a worst case exponential overhead of $\mathcal{ALC}_{\mathcal{B}}$-formulas.

Let us now reconsider example 3.9: Given the $\mathcal{ALC}_{\mathcal{B}}$-formulas $\Diamond_1 F_1$, $\Diamond_1 F_2$, and $\Diamond_1 F_3$ the optimized KD45-satisfiability algorithm constructs the $\mathcal{ALC}$ test sets $\{F_1\}$,

1. Let $\mathsf{S}_1 F_1, \ldots, \mathsf{S}_n F_n$ be the $\mathcal{ALC}_{\mathcal{B}}$-formulas to be tested on KD45-satisfiability, where each S_i is a (possibly empty) sequence of modalities and each F_i is an $\mathcal{ALC}$-formula.

2. for $i := 1, \ldots, n$ do
   if $F_i \in S_k$ for some $k \in \{1, \ldots, i-1\}$
   then $S_i := \emptyset$
   else
     $S_i := \{F_i\}$
     for $j := i+1, \ldots, n$ do
       if $indexes(\mathsf{S}_i) = indexes(\mathsf{S}_j)$ and
         there is no position k such that $\mathsf{S}_i[k] = \mathsf{S}_i[k] = \Diamond_l$ for some agent l
       then $S_i := S_i \cup \{F_j\}$
     endfor
   endfor

3. For each non-empty set s in $\{S_1, \ldots, S_n\}$ do: If s is not satisfiable, then STOP and return "KD45-unsatisfiable".

4. Return "KD45-satisfiable".

Figure 3: The optimized KD45-satisfiability algorithm.

$\{F_2\}$, and $\{F_3\}$ without applying the frame algorithm. Analogously, from $\Box_1(A = C)$ and $\Box_1(A = D)$ the only $\mathcal{ALC}$ test set $\{A = C, A = D\}$ is generated immediately from the syntactical structure of the input $\mathcal{ALC}_{\mathcal{B}}$-formulas (cf. example 4.1).

4.2 Restricted $\mathcal{ALC}$-TBoxes

In Section 2 we defined an $\mathcal{ALC}$-TBox as a set of terminological axioms of the form $C = D$ and $C \neq D$, where C and D are concepts. However, most of the existing terminological representation and inference systems (e.g., BACK [30], CLASSIC [7], $\mathcal{KRIS}$ [2]) only allow terminological axioms of the form $A = C$, where A is a primitive concept and C is a concept. Such a terminological axiom is called *(concept) definition* of A.[6] Building upon this, an $\mathcal{ALC}$-TBox is then defined as a finite set of terminological axioms which satisfies the following restrictions:

- each atomic concept appears at most once as the left hand side of a terminological axiom, and

[6] Often so-called concept specializations of the form $A \sqsubseteq C$ are allowed which abbreviate the terminological axiom $A = C \sqcap A^*$ where A^* is a new primitive concept.

- in this set cycles do not occur.

Thereby, a set $\mathcal{S}$ of terminological axioms contains a *cycle* iff there exists a terminological axiom $A = C$ in $\mathcal{S}$ such that A occurs in the concept C' which arises from C by iterated substitutions of primitive concepts in C by the right hand sides of their definition in $\mathcal{S}$. For example, if A and B are primitive concepts the sets $\{A = A\}$ and $\{A = C \sqcap B, B = D \sqcup \exists R.A\}$ of terminological axioms contain cycles. In the following we will call $\mathcal{ALC}$-TBoxes satisfying the additional conditions described above *restricted $\mathcal{ALC}$-TBoxes* in order to distinguish them from the $\mathcal{ALC}$-TBoxes defined in Section 2.

It can be shown that each restricted $\mathcal{ALC}$-TBox $\mathcal{T}$ can be transformed into an equivalent restricted $\mathcal{ALC}$-TBox $\mathcal{T}'$ such that each right hand side of a concept definition in $\mathcal{T}'$ does only contain concepts which do not occur as a left hand side in $\mathcal{T}'$ (see, e.g., [20]). For example, if A_1, A_2, A_3 are primitive concepts, the restricted $\mathcal{ALC}$-TBox $\mathcal{T} = \{A_1 = A_2 \sqcap A_3, A_2 = C \sqcup D, A_3 = \exists R.C\}$ can be transformed into $\mathcal{T}' = \{A_1 = (C \sqcup D) \sqcap \exists R.C, A_2 = C \sqcup D, A_3 = \exists R.C\}$. Thus, each primitive concept A on the left hand side of a terminological axiom $A = C$ in $\mathcal{T}'$ can be seen as an abbreviation for the concept C. With this it is easy to verify that testing consistency of an $\mathcal{ALC}$-ABox $\mathcal{A}$ w.r.t. a restricted $\mathcal{ALC}$-TBox $\mathcal{T}$ is equivalent to only testing consistency of an $\mathcal{ALC}$-ABox $\mathcal{A}'$. Thereby, $\mathcal{A}'$ arises from $\mathcal{A}$ by successively replacing all primitive concepts by the right hand sides of their definitions in $\mathcal{T}$. The size of $\mathcal{A}'$ is worst case exponential in in the size of $\mathcal{A}$ and $\mathcal{T}$ (see, e.g., [20]) and testing satisfiability of $\mathcal{A}'$ is known to be PSPACE-complete (see [19]). Possible optimizations are discussed in [1].

An algorithm for testing consistency of $\mathcal{ALC}$ test sets which may contain terminological axioms as defined in Section 2 has been given in [21]. As an easy consequence of a result by Fischer and Ladner [13] this test is EXP-TIME complete. Moreover, when using more expressive terminological logics than $\mathcal{ALC}$ this test becomes undecidable (for the terminological logic $\mathcal{ALCF}$ this has been shown in [4]), while this is not the case when using restricted $\mathcal{ALC}$-TBoxes only.

Let now $\mathcal{S}$ be a set of $\mathcal{ALC}_{\mathcal{B}}$-formulas. Because of the above given discussion on efficiency and decidability of testing satisfiability of an $\mathcal{ALC}$-TBox and an $\mathcal{ALC}$-ABox it is an interesting question whether or not the terminological axioms in each $\mathcal{ALC}$ test set which is generated from $\mathcal{S}$ define a restricted $\mathcal{ALC}$-TBox. The answer to this question can be given with the help of Theorem 4.6 which can be used to formulate sufficient syntactical conditions which—if satisfied—guarantee that only restricted $\mathcal{ALC}$-TBoxes have to be tested in order to test KD45-satisfiability of a set of $\mathcal{ALC}_{\mathcal{B}}$-formulas. For example, these conditions could be given by

1. agents only have positive beliefs, i.e., negation signs do not occur in front of $\Box$-operators, and

2. for each sequence S of modalities holds that the set of $\mathcal{ALC}_{\mathcal{B}}$-formulas occurring in the scope of S define a restricted $\mathcal{ALC}$-TBox.

For practical applications, however, such conditions seem not reasonable and, even worse, when computing logical consequences (see next subsection) such syntactical conditions in general cannot be maintained. Hence, for testing satisfiability of $\mathcal{ALC}$ test sets we in general have to take terminological axioms into account which do not define a restricted $\mathcal{ALC}$-TBox.

4.3 Computing $\mathcal{ALC}_{\mathcal{B}}$-Inferences

We will now show how to use the KD45-satisfiability algorithm in order to test whether or not a given $\mathcal{ALC}_{\mathcal{B}}$-formula is a logical consequence from a set $F_1, \ldots, F_n$ of $\mathcal{ALC}_{\mathcal{B}}$-formulas. Again, we are only interested in KD45 Kripke structures and thus define: F is a *KD45 consequence* of $F_1, \ldots, F_n$ iff for each KD45 Kripke structure $K = (\mathcal{W}, \Gamma, K_I)$ and for each world w in $\mathcal{W}$ holds: if $K, w \models F_1, \ldots, F_n$, then $K, w \models F$. Firstly, let F be an $\mathcal{ALC}_{\mathcal{B}}$-formula of the form $\circ^*(C = D)$, $\circ^*(C \neq D)$, or $\circ^*(a : C)$, where $\circ^*$ is an abbreviation for a (possibly empty) sequence of modalities. Then, F is an KD45 consequence of $F_1, \ldots, F_n$ iff the set $F_1, \ldots, F_n, [\neg F]^*$ of $\mathcal{ALC}_{\mathcal{B}}$-formulas is not KD45-satisfiable, where $[\neg F]^*$ denotes the negation normal form of $\neg F$. Note, that $\neg F$ is an $\mathcal{ALC}_{\mathcal{B}}$-formula if F is of the above described form.

If, on the other hand, F is of the form $\Box^*(aRb)$, where $\Box^*$ is an abbreviation for a (possibly empty) sequence of non-negated indexed $\Box$ operators, we cannot use this test method since negation signs are not allowed in $\mathcal{ALC}_{\mathcal{B}}$-formulas which contain a role instance. To handle this case, we extend the notion of $\mathcal{ALC}_{\mathcal{K}}$-formulas as follows: if R is a role, a, b are objects, and $i_1, \ldots, i_m$ are agents, then $\Diamond_{i_1} \ldots \Diamond_{i_m}(aRb)$ is an $\mathcal{ALC}_{\mathcal{B}}$-formula.

Alternatively, these $\mathcal{ALC}_{\mathcal{B}}$-formulas could be defined by $\circ_{i_1} \ldots \circ_{i_m} (aR'b)$ where (i) each $\circ_{i_j}$ is either $\Box_{i_j}$ or $\neg\Box_{i_j}$, (ii) R' is either R or $\neg R$, and (iii) the number of negation signs in $\circ_{i_1} \ldots \circ_{i_m} (aR'b)$ is even. Using this definition it is easy to see that the negation normal form of the new $\mathcal{ALC}_{\mathcal{B}}$-formulas does not contain negation of roles. Therefore, on a technical level we could allow such formulas as $\mathcal{ALC}_{\mathcal{K}}$-axioms in Section 2. But a restriction like "the number of negation signs is even" seems not to be adequate when defining a language to describe beliefs of agents. However, for testing whether or not an $\mathcal{ALC}_{\mathcal{B}}$-formula is entailed by a set $F_1, \ldots, F_n$ of $\mathcal{ALC}_{\mathcal{B}}$-axioms, this definition turns out to be reasonable.

Note, that KD45-satisfiability of a set of $\mathcal{ALC}_{\mathcal{B}}$-formulas can be handled by the KD45-satisfiability algorithm in Section 3 even if we use the above introduced extended definition of $\mathcal{ALC}_{\mathcal{B}}$-formulas: Firstly, the algorithm only treats the modalities of $\mathcal{ALC}_{\mathcal{B}}$-formulas, i.e., it works independently from the syntactical structure of formulas without modalities. Secondly, satisfiability of the resulting $\mathcal{ALC}$ test sets still can be tested, since they do not contain negation of roles. And, finally, it does not matter whether aRb is in an $\mathcal{ALC}$ test set because of an input $\mathcal{ALC}_{\mathcal{B}}$-formula $\Box_{i_1} \ldots \Box_{i_m}(aRb)$, or because of an input $\mathcal{ALC}_\mathcal{B}$-formula $\Diamond_{i_1} \ldots \Diamond_{i_m}(aRb)$. Summing up, when using the extended definition of $\mathcal{ALC}_\mathcal{B}$-formulas we need not to change the KD45-satisfiability algorithm at all. The following proposition provides a test whether or not an $\mathcal{ALC}_\mathcal{B}$-formula $\Box_{i_1} \ldots \Box_{i_n}(aRb)$ is entailed by a set of $\mathcal{ALC}_\mathcal{B}$-formulas.

Proposition 4.7 *Let $F_1, \ldots, F_n$ be a finite set of $\mathcal{ALC}_\mathcal{B}$-formulas such that $F_1, \ldots, F_n$ are KD45-satisfiable, let a and b be objects, let R be a role, and let $i_1, \ldots, i_m$ be agents. Then $\Box_{i_1} \ldots \Box_{i_m}(aRb)$ is a KD45 consequence of $F_1, \ldots, F_n$ iff $\Box_{i_1} \ldots \Box_{i_m}(aRb)$ is one of the $\mathcal{ALC}_\mathcal{B}$-formulas in $F_1, \ldots, F_n$.*

Proof: The test whether or not $\Box_{i_1} \ldots \Box_{i_m}(aRb)$ is a KD45 consequence of $F_1, \ldots, F_n$ is equivalent to testing whether or not $F_1, \ldots, F_n, \Diamond_{i_1} \ldots \Diamond_{i_m}(a\neg Rb)$ is KD45-satisfiable. However, since $\Diamond_{i_1} \ldots \Diamond_{i_m}(a\neg Rb)$ is not an $\mathcal{ALC}_\mathcal{B}$-formula, this case cannot be handled by the KD45-satisfiability algorithm of Section 3. Alternatively, let us have a look at the application of the frame algorithm to the world constraint system W which is induced by $\{F_1, \ldots, F_n, \Diamond_{i_1} \ldots \Diamond_{i_m}(aR'b)\}$, where R' is a role which does not occur in $F_1, \ldots, F_n$. Since $F_1, \ldots, F_n$ are KD45-satisfiable, each $\mathcal{ALC}$ test set is satisfiable which is constructed by applying the frame algorithm to the world constraint system which is induced by $\{F_1, \ldots, F_n\}$. Hence, it is easy to verify that the $\mathcal{ALC}_\mathcal{B}$-formulas $F_1, \ldots, F_n, \Diamond_{i_1} \ldots \Diamond_{i_m}(aR'b)$ are KD45-satisfiable.

Let us now consider R' as an abbreviation for $\neg R$. Obviously, this does not influence the result of applying the frame algorithm to W, but it may influence satisfiability of the thereby computed $\mathcal{ALC}$ test sets—say $A(w_1), \ldots, A(w_k)$—in which $aR'b$ occurs. It is easy to verify that, given an $\mathcal{ALC}$-TBox $\mathcal{T}$ and an $\mathcal{ALC}$-ABox $\mathcal{A}$, the $\mathcal{ALC}$-formula aRb is logically entailed by $\mathcal{T}$ and $\mathcal{A}$ iff $aRb \in \mathcal{A}$. Hence, an $\mathcal{ALC}$ test set $A(w_i) \in \{A(w_1), \ldots, A(w_k)\}$ is unsatisfiable iff aRb and $aR'b$ are both in $A(w_i)$. Because of Theorem 4.6 the $\mathcal{ALC}$-formulas aRb and $aR'b$ can only be in the same $\mathcal{ALC}$ test set if there is a labeled $\mathcal{ALC}_\mathcal{B}$-formula $\Box_{i_1} \ldots \Box_{i_m}(aRb)$ in W. From this the proposition follows immediately. □

Summing up, we have now given algorithms for deciding KD45-satisfiability of a given set of $\mathcal{ALC}_\mathcal{B}$-formulas, and, building upon this, for deciding whether or not a given $\mathcal{ALC}_\mathcal{B}$-formula F is a KD45 consequence of a given set $F_1, \ldots, F_n$ of $\mathcal{ALC}_\mathcal{B}$-formulas.

5 Conclusion

We have presented a two-dimensional extension of the concept language $\mathcal{ALC}$ which allows both reasoning on the objective level and reasoning on the level of epistemic alternatives. In the obtained language $\mathcal{ALC}_\mathcal{B}$, a world agents are acting in can be described by a set of terminological and assertional axioms. Furthermore, the beliefs agents have about this world, about the beliefs of other agents, and about their own beliefs can be described by terminological and assertional axioms with a leading indexed $\Box$ operator or a leading sequence of indexed $\Box$ operators. We presented sound and complete algorithms to check consistency of the represented beliefs and to decide whether an $\mathcal{ALC}_\mathcal{B}$-formula is logically entailed by a given set of $\mathcal{ALC}_\mathcal{B}$-formulas. Thus, it is possible to equip agents with a decidable component to represent beliefs that is much more expressive than representing beliefs via propositional logic. Since the actions a single agent can perform are essentially based on his local beliefs this component can be seen as one of the basic parts in the architecture of agents.

The main restriction of the presented language $\mathcal{ALC}_\mathcal{B}$ lies in the fact that modalities are only allowed in front of terminological and assertional axioms. As an extension one might think of modalities in front of concepts as well. Such a language would allow to represent facts like "the things agent i believes to be expensive are exactly the things agent j believes to be cheap" by

$$\Box_i(\mathit{expensive}) = \Box_j(\mathit{cheap}).$$

Such an extended language, however, causes algorithmic problems that are beyond the scope of this paper and is currently investigated.

Acknowledgements. I am grateful to Hans-Jürgen Bürckert, Bernhard Hollunder, and Andreas Nonnengart for many discussions on the topic of this paper and for reading earlier drafts.

References

[1] F. Baader, E. Franconi, B. Hollunder, B. Nebel, and H.-J. Profitlich. An empirical analysis of optimization techniques for terminological representation systems or: Making $\mathcal{KRIS}$ get a move on. In *Proceedings of the 3rd International Conference on Knowledge Representation and Reasoning*, Cambridge, Mass., 1992.

[2] F. Baader and B. Hollunder. $\mathcal{KRIS}$: $\mathcal{K}$nowledge $\mathcal{R}$epresentation and $\mathcal{I}$nference $\mathcal{S}$ystem. *SIGART Bulletin*, 2(3):8–14, 1991.

[3] F. Baader and B. Hollunder. A terminological knowledge representation system with complete inference algorithms. In M. Richter and H. Boley, editors, *International Workshop on Processing Declarative Knowledge*, 1991.

[4] F. Baader and B. Hollunder. Embedding defaults into terminological knowledge representation formalisms. In *Proceedings of the 3rd International Conference on Knowledge Representation and Reasoning*, Cambridge, Mass., 1992.

[5] F. Baader and H.-J. Ohlbach. A multi-dimensional terminological knowledge representation language. In *Proceedings of IJCAI 93*, 1993. To appear.

[6] A. Bond and L. Gasser. *Readings in Distributed Artificial Intelligence.* Morgan Kaufmann, Los Angeles, CA, 1988.

[7] A. Borgida, R. J. Brachman, D. L. McGuinness, and L. A. Resnick. CLASSIC: a structural data model for objects. In *Proceedings of the 1989 ACM SIGMOD International Conference on Management of Data*, pages 59–67, Portland, Oreg., 1989.

[8] R. Brachman and H. J. Levesque. Expressiveness and tractability in knowledge representation and reasoning. *Computational Intelligence*, 3:78–93, 1987.

[9] R. J. Brachman and J. G. Schmolze. An overwiev of the KL-ONE knowledge representation system. *Cognitive Science*, 9(2):171–216, 1985.

[10] B. F. Chellas. *Modal Logic: An Introduction.* Cambridge University Press, 1980.

[11] N. Dershowitz and Z. Manna. Proving termination with multiset orderings. *ACM*, 22(8):465–476, 1979.

[12] R. Fagin and J. Y. Halpern. Two views of belief. *Artificial Intelligence*, 54(3):275–317, April 1992.

[13] M. J. Fischer and R. E. Ladner. Propositional dynamic logic of regular programs. *Journal of Computer and System Science*, 18:194–211, 1979.

[14] M. Fitting. *Proof Methods for Modal and Intuitionistic Logics*, volume 169 of *Synthese Library.* D. Reidel Publishing Company, 1983.

[15] L. Gasser and M.N. Huhns. *Distributed Artificial Intelligence, Volume II.* Research Notes in Artificial Intelligence. Morgan Kaufmann, San Mateo, CA, 1989.

[16] J. Y. Halpern and Y. Moses. Knowledge and common knowledge in a distributed environment. *Journal of the ACM*, 37(3):549–587, 1990.

[17] J. Y. Halpern and Y. Moses. A guide to completeness and complexity for modal logics of knowledge and belief. *Artificial Intelligence*, 54:319–379, 1992.

[18] J. Hintikka, editor. *Knowledge and Belief.* Cornell University Press, 1962.

[19] B. Hollunder. How to reduce reasoning to satisfiability checking of concepts in the terminological system $\mathcal{KRIS}$. To appear.

[20] B. Hollunder. Hybrid inferences in KL-ONE-based knowledge representation systems. In *14th German Workshop on Artificial Intelligence*, volume 251 of *Informatik-Fachberichte*, pages 38–47, Ebingerfeld, Germany, 1990. Springer.

[21] A. Laux. Integrating a modal logic of knowledge into terminological logics. Research Report RR-92-56, DFKI Saarbrücken, 1992.

[22] W. Lenzen. Recent work in epistemic logic. *Acta Philosophica Fennica*, 30:1–219, 1978.

[23] H. J. Levesque. Foundations of a functional approach to knowledge representation. *Artificial Intelligence*, 23:155–212, 1984.

[24] G. L. McArthur. Reasoning about knowledge and belief: A survey. *Computational Intelligence*, 4:223–243, 1988.

[25] J.-J. C. Meyer, W. van der Hoek, and G. A. W. Vreeswijk. Epistemic logic for computer science: A tutorial (part one). In *Bulletin of the EATCS*, volume 44, pages 242–270. European Association for Theoretical Computer Science, 1991.

[26] J.-J. C. Meyer, W. van der Hoek, and G. A. W. Vreeswijk. Epistemic logic for computer science: A tutorial (part two). In *Bulletin of the EATCS*, volume 45, pages 256–287. European Association for Theoretical Computer Science, 1991.

[27] R. C. Moore. Reasoning about knowledge and action. Technical Report 191, SRI International, 1980.

[28] R. C. Moore. A formal theory of knowledge and action. In J. R. Hobbs and R. C. Moore, editors, *Formal Theories of the Commonsense World*, pages 319–358. Ablex Publishing Corporation, 1985.

[29] B. Nebel. *Reasoning and Revision in Hybrid Knowledge Representation Systems.* Number 422 in LNAI. Springer, 1990.

[30] B. Nebel and K. von Luck. Hybrid reasoning in BACK. In Z. W. Ras and L. Saitta, editors, *Methodologies for Intelligent Systems*, volume 3, pages 260–269. North-Holland, 1988.

[31] K. Schild. A correspondence theory for terminological logics: Preliminary report. In *Proceedings of IJCAI 91*, pages 466–471, 1991.

Deutsches
Forschungszentrum
für Künstliche
Intelligenz GmbH

DFKI
-Bibliothek-
PF 2080
67608 Kaiserslautern
FRG

DFKI Publikationen

Die folgenden DFKI Veröffentlichungen sowie die aktuelle Liste von allen bisher erschienenen Publikationen können von der oben angegebenen Adresse bezogen werden.
Die Berichte werden, wenn nicht anders gekennzeichnet, kostenlos abgegeben.

DFKI Publications

The following DFKI publications or the list of all published papers so far can be ordered from the above address.
The reports are distributed free of charge except if otherwise indicated.

DFKI Research Reports

RR-92-42
John Nerbonne:
A Feature-Based Syntax/Semantics Interface
19 pages

RR-92-43
Christoph Klauck, Jakob Mauss: A Heuristic driven Parser for Attributed Node Labeled Graph Grammars and its Application to Feature Recognition in CIM
17 pages

RR-92-44
Thomas Rist, Elisabeth André: Incorporating Graphics Design and Realization into the Multimodal Presentation System WIP
15 pages

RR-92-45
Elisabeth André, Thomas Rist: The Design of Illustrated Documents as a Planning Task
21 pages

RR-92-46
Elisabeth André, Wolfgang Finkler, Winfried Graf, Thomas Rist, Anne Schauder, Wolfgang Wahlster:
WIP: The Automatic Synthesis of Multimodal Presentations
19 pages

RR-92-47
Frank Bomarius: A Multi-Agent Approach towards Modeling Urban Traffic Scenarios
24 pages

RR-92-48
Bernhard Nebel, Jana Koehler:
Plan Modifications versus Plan Generation: A Complexity-Theoretic Perspective
15 pages

RR-92-49
Christoph Klauck, Ralf Legleitner, Ansgar Bernardi:
Heuristic Classification for Automated CAPP
15 pages

RR-92-50
Stephan Busemann:
Generierung natürlicher Sprache
61 Seiten

RR-92-51
Hans-Jürgen Bürckert, Werner Nutt:
On Abduction and Answer Generation through Constrained Resolution
20 pages

RR-92-52
Mathias Bauer, Susanne Biundo, Dietmar Dengler, Jana Koehler, Gabriele Paul: PHI - A Logic-Based Tool for Intelligent Help Systems
14 pages

RR-92-53
Werner Stephan, Susanne Biundo:
A New Logical Framework for Deductive Planning
15 pages

RR-92-54
Harold Boley: A Direkt Semantic Characterization of RELFUN
30 pages

RR-92-55
John Nerbonne, Joachim Laubsch, Abdel Kader Diagne, Stephan Oepen: Natural Language Semantics and Compiler Technology
17 pages

RR-92-56
Armin Laux: Integrating a Modal Logic of Knowledge into Terminological Logics
34 pages

RR-92-58
Franz Baader, Bernhard Hollunder:
How to Prefer More Specific Defaults in Terminological Default Logic
31 pages

RR-92-59
Karl Schlechta and David Makinson: On Principles and Problems of Defeasible Inheritance
13 pages

RR-92-60
Karl Schlechta: Defaults, Preorder Semantics and Circumscription
19 pages

RR-93-02
Wolfgang Wahlster, Elisabeth André, Wolfgang Finkler, Hans-Jürgen Profitlich, Thomas Rist: Plan-based Integration of Natural Language and Graphics Generation
50 pages

RR-93-03
Franz Baader, Berhard Hollunder, Bernhard Nebel, Hans-Jürgen Profitlich, Enrico Franconi: An Empirical Analysis of Optimization Techniques for Terminological Representation Systems
28 pages

RR-93-04
Christoph Klauck, Johannes Schwagereit: GGD: Graph Grammar Developer for features in CAD/CAM
13 pages

RR-93-05
Franz Baader, Klaus Schulz: Combination Techniques and Decision Problems for Disunification
29 pages

RR-93-06
Hans-Jürgen Bürckert, Bernhard Hollunder, Armin Laux: On Skolemization in Constrained Logics
40 pages

RR-93-07
Hans-Jürgen Bürckert, Bernhard Hollunder, Armin Laux: Concept Logics with Function Symbols
36 pages

RR-93-08
Harold Boley, Philipp Hanschke, Knut Hinkelmann, Manfred Meyer: COLAB: A Hybrid Knowledge Representation and Compilation Laboratory
64 pages

RR-93-09
Philipp Hanschke, Jörg Würtz: Satisfiability of the Smallest Binary Program
8 Seiten

RR-93-10
Martin Buchheit, Francesco M. Donini, Andrea Schaerf: Decidable Reasoning in Terminological Knowledge Representation Systems
35 pages

RR-93-11
Bernhard Nebel, Hans-Juergen Buerckert: Reasoning about Temporal Relations: A Maximal Tractable Subclass of Allen's Interval Algebra
28 pages

RR-93-12
Pierre Sablayrolles: A Two-Level Semantics for French Expressions of Motion
51 pages

RR-93-13
Franz Baader, Karl Schlechta: A Semantics for Open Normal Defaults via a Modified Preferential Approach
25 pages

RR-93-14
Joachim Niehren, Andreas Podelski, Ralf Treinen: Equational and Membership Constraints for Infinite Trees
33 pages

RR-93-15
Frank Berger, Thomas Fehrle, Kristof Klöckner, Volker Schölles, Markus A. Thies, Wolfgang Wahlster: PLUS - Plan-based User Support Final Project Report
33 pages

RR-93-16
Gert Smolka, Martin Henz, Jörg Würtz: Object-Oriented Concurrent Constraint Programming in Oz
17 pages

RR-93-17
Rolf Backofen: Regular Path Expressions in Feature Logic
37 pages

RR-93-18
Klaus Schild: Terminological Cycles and the Propositional μ-Calculus
32 pages

RR-93-20
Franz Baader, Bernhard Hollunder: Embedding Defaults into Terminological Knowledge Representation Formalisms
34 pages

RR-93-22
Manfred Meyer, Jörg Müller: Weak Looking-Ahead and its Application in Computer-Aided Process Planning
17 pages

RR-93-23
Andreas Dengel, Ottmar Lutzy: Comparative Study of Connectionist Simulators
20 pages

RR-93-24
Rainer Hoch, Andreas Dengel: Document Highlighting — Message Classification in Printed Business Letters
17 pages

RR-93-25
Klaus Fischer, Norbert Kuhn: A DAI Approach to Modeling the Transportation Domain
93 pages

RR-93-26
Jörg P. Müller, Markus Pischel: The Agent Architecture InteRRaP: Concept and Application
99 pages

RR-93-27
Hans-Ulrich Krieger:
Derivation Without Lexical Rules
33 pages

RR-93-28
Hans-Ulrich Krieger, John Nerbonne, Hannes Pirker: Feature-Based Allomorphy
8 pages

RR-93-29
Armin Laux: Representing Belief in Multi-Agent Worlds viaTerminological Logics
35 pages

RR-93-33
Bernhard Nebel, Jana Koehler:
Plan Reuse versus Plan Generation: A Theoretical and Empirical Analysis
33 pages

RR-93-34
Wolfgang Wahlster:
Verbmobil Translation of Face-To-Face Dialogs
10 pages

RR-93-35
Harold Boley, François Bry, Ulrich Geske (Eds.):
Neuere Entwicklungen der deklarativen KI-Programmierung — *Proceedings*
150 Seiten
Note: This document is available only for a nominal charge of 25 DM (or 15 US-S).

RR-93-36
Michael M. Richter, Bernd Bachmann, Ansgar Bernardi, Christoph Klauck, Ralf Legleitner, Gabriele Schmidt: Von IDA bis IMCOD: Expertensysteme im CIM-Umfeld
13 Seiten

RR-93-38
Stephan Baumann: Document Recognition of Printed Scores and Transformation into MIDI
24 pages

RR-93-40
Francesco M. Donini, Maurizio Lenzerini, Daniele Nardi, Werner Nutt, Andrea Schaerf:
Queries, Rules and Definitions as Epistemic Statements in Concept Languages
23 pages

RR-93-41
Winfried H. Graf: LAYLAB: A Constraint-Based Layout Manager for Multimedia Presentations
9 pages

RR-93-42
Hubert Comon, Ralf Treinen:
The First-Order Theory of Lexicographic Path Orderings is Undecidable
9 pages

RR-93-45
Rainer Hoch: On Virtual Partitioning of Large Dictionaries for Contextual Post-Processing to Improve Character Recognition
21 pages

DFKI Technical Memos

TM-91-15
Stefan Busemann: Prototypical Concept Formation
An Alternative Approach to Knowledge Representation
28 pages

TM-92-01
Lijuan Zhang: Entwurf und Implementierung eines Compilers zur Transformation von Werkstückrepräsentationen
34 Seiten

TM-92-02
Achim Schupeta: Organizing Communication and Introspection in a Multi-Agent Blocksworld
32 pages

TM-92-03
Mona Singh:
A Cognitiv Analysis of Event Structure
21 pages

TM-92-04
Jürgen Müller, Jörg Müller, Markus Pischel, Ralf Scheidhauer:
On the Representation of Temporal Knowledge
61 pages

TM-92-05
Franz Schmalhofer, Christoph Globig, Jörg Thoben:
The refitting of plans by a human expert
10 pages

TM-92-06
Otto Kühn, Franz Schmalhofer: Hierarchical skeletal plan refinement: Task- and inference structures
14 pages

TM-92-08
Anne Kilger: Realization of Tree Adjoining Grammars with Unification
27 pages

TM-93-01
Otto Kühn, Andreas Birk: Reconstructive Integrated Explanation of Lathe Production Plans
20 pages

TM-93-02
Pierre Sablayrolles, Achim Schupeta:
Conlfict Resolving Negotiation for COoperative Schedule Management
21 pages

TM-93-03
Harold Boley, Ulrich Buhrmann, Christof Kremer:
Konzeption einer deklarativen Wissensbasis über recyclingrelevante Materialien
11 pages

TM-93-04
Hans-Günther Hein: Propagation Techniques in WAM-based Architectures — The FIDO-III Approach
105 pages

DFKI Documents

D-92-19
Stefan Dittrich, Rainer Hoch: Automatische, Deskriptor-basierte Unterstützung der Dokument-analyse zur Fokussierung und Klassifizierung von Geschäftsbriefen
107 Seiten

D-92-21
Anne Schauder: Incremental Syntactic Generation of Natural Language with Tree Adjoining Grammars
57 pages

D-92-22
Werner Stein: Indexing Principles for Relational Languages Applied to PROLOG Code Generation
80 pages

D-92-23
Michael Herfert: Parsen und Generieren der Prolog-artigen Syntax von RELFUN
51 Seiten

D-92-24
Jürgen Müller, Donald Steiner (Hrsg.): Kooperierende Agenten
78 Seiten

D-92-25
Martin Buchheit: Klassische Kommunikations- und Koordinationsmodelle
31 Seiten

D-92-26
Enno Tolzmann: Realisierung eines Werkzeugauswahlmoduls mit Hilfe des Constraint-Systems CONTAX
28 Seiten

D-92-27
Martin Harm, Knut Hinkelmann, Thomas Labisch: Integrating Top-down and Bottom-up Reasoning in COLAB
40 pages

D-92-28
Klaus-Peter Gores, Rainer Bleisinger: Ein Modell zur Repräsentation von Nachrichtentypen
56 Seiten

D-93-01
Philipp Hanschke, Thom Frühwirth: Terminological Reasoning with Constraint Handling Rules
12 pages

D-93-02
Gabriele Schmidt, Frank Peters, Gernod Laufkötter: User Manual of COKAM+
23 pages

D-93-03
Stephan Busemann, Karin Harbusch(Eds.): DFKI Workshop on Natural Language Systems: Reusability and Modularity - Proceedings
74 pages

D-93-04
DFKI Wissenschaftlich-Technischer Jahresbericht 1992
194 Seiten

D-93-05
Elisabeth André, Winfried Graf, Jochen Heinsohn, Bernhard Nebel, Hans-Jürgen Profitlich, Thomas Rist, Wolfgang Wahlster: PPP: Personalized Plan-Based Presenter
70 pages

D-93-06
Jürgen Müller (Hrsg.): Beiträge zum Gründungsworkshop der Fachgruppe Verteilte Künstliche Intelligenz Saarbrücken 29.-30. April 1993
235 Seiten
Note: This document is available only for a nominal charge of 25 DM (or 15 US-$).

D-93-07
Klaus-Peter Gores, Rainer Bleisinger: Ein erwartungsgesteuerter Koordinator zur partiellen Textanalyse
53 Seiten

D-93-08
Thomas Kieninger, Rainer Hoch: Ein Generator mit Anfragesystem für strukturierte Wörterbücher zur Unterstützung von Texterkennung und Textanalyse
125 Seiten

D-93-09
Hans-Ulrich Krieger, Ulrich Schäfer: TDL ExtraLight User's Guide
35 pages

D-93-10
Elizabeth Hinkelman, Markus Vonerden,Christoph Jung: Natural Language Software Registry (Second Edition)
174 pages

D-93-11
Knut Hinkelmann, Armin Laux (Eds.): DFKI Workshop on Knowledge Representation Techniques — Proceedings
88 pages

D-93-12
Harold Boley, Klaus Elsbernd, Michael Herfert, Michael Sintek, Werner Stein: RELFUN Guide: Programming with Relations and Functions Made Easy
86 pages

D-93-14
Manfred Meyer (Ed.): Constraint Processing – Proceedings of the International Workshop at CSAM'93, July 20-21, 1993
264 pages
Note: This document is available only for a nominal charge of 25 DM (or 15 US-$).

Representing Belief in Multi-Agent Worlds via Terminological Logics

Armin Laux

RR-93-29

Research Report